Universal Multiplicity Distributions in pp-and Heavy-Ion Collisions

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Outline

- Maximum Entropy approach to a nonthermal plasma
- Determining the parameters of the universal distribution for pp-Collisions
- Generalizing the universal distribution for Heavy-Ion Collisions
- Explaining the Heavy-Ion transverse momentum broadening from parton rescattering (saturation)

The Maximum Entropy Approach

the maximum-entropy distribution. The dynamics of collisions at high energies is governed by a the light-cone Hamiltonian which is boost invariant and determines wavefunctions depending on transverse momentum and light-cone fractions which lable the eigenstates. The density matrix resulting after the collision can be built up from a incoherent mixture of such multi-parton states. Without a boost invariant formulation one cannot define a number density of partons since in each reference system it will be different. In the rest system of

Transverse Momentum and Light cone fractions

$$x = \frac{\epsilon + p_z}{E + P_z} = \frac{p_+}{P_+}.$$

of gluons-for simplification

The Maximum Entropy Approach

Each cell is labeled by impact parameter, transverse momentum, light cone momentum and longitudinal light cone coordinate. The number of quantum states in each cell is then given by the volume of the cell divided by h^3 and multiplied by the degeneracy factor for gluons g=2*8 :

$$\hat{G}_{b_\perp, p_\perp, p_+, x_-} = g \frac{d^2 b_\perp d^2 p_\perp d p_+ d x_-}{(2\pi)^3} = g \frac{d^2 b_\perp d^2 p_\perp}{(2\pi)^2} dx \frac{d\rho}{2\pi}$$

$$x = dp_+ / P_+$$
$$\rho = dx_- P_+$$

Integrate out coordinate space:

Transverse space : $L_\perp^2$

Longitudinal space: $\int \frac{d\rho}{2\pi} \approx \frac{1}{x}$

homogeneously distributed

$$\int \frac{d\rho_{val}}{2\pi} \approx \frac{L_z}{\gamma}\frac{m\gamma}{2\pi} \approx 1$$

$$G_{x,p_\perp} = gL_\perp^2 \frac{d^2 p_\perp}{(2\pi)^2}\frac{dx}{x}.$$

$$\int \frac{d\rho_{sea}}{2\pi} \approx \frac{1}{xP} P \to \infty.$$

The Maximum Entropy Approach

We consider a fixed cell: Then to get to the entropy, we count the number of possibilities to distribute N particles on G quantum states , allowing multiple occupancy of each quantum state, because gluons are bosons.

Take e.g. N=11 and G=4 : o o o | o o o o o | o | o o

$$\Delta\Gamma_{x,p_\perp} = \frac{(G_{x,p_\perp} + N_{x,p_\perp} - 1)!}{(G_{x,p_\perp} - 1)!N_{x,p_\perp}!}$$

The entropy

The entropy is given by the
log of the total phase space.
Use Stirlings formula and factor out G:

$$S = \sum \ln(\Delta\Gamma_{x,p_\perp}) \tag{4}$$

$$= \sum G_{x,p_\perp}[(1+n_{x,p_\perp})\ln(1+n_{x,p_\perp}) - n_{x,p_\perp}\ln(n_{x,p_\perp})]. \tag{5}$$

We want to maximize this entropy under the following two-constraints

$$\sum G_{x,p_\perp} |p_\perp| n_{x,p_\perp} = \langle E_\perp \rangle.$$

$$\sum G_{x,p_\perp} x n_{x,p_\perp} = 1$$

The total transverse energy is fixed

The light cone fractions of all partons (gluons) add up to unity

Search for extremum of S+ 1/λ E(transverse)+w unity :

The Lagrange parameters are the transverse temperature λ and the softness w

$$\frac{\delta(S + \frac{1}{\lambda}\sum |p_\perp| n_{x,p_\perp} + w \sum x n_{x,p_\perp})}{\delta n_{x,p_\perp}} = 0.$$

Result: The Light Cone Plasma Distribution

$$n_{x,p_\perp} = \frac{1}{e^{\frac{|\vec{p}_\perp|}{\lambda}+xw} - 1}$$

We expect that with increasing cm energy the gluons on the light cone will get hotter and softer, i.e. their transverse temperature and softness will increase , i.e. their x-fractions will become smaller

Input parameters L,λ,w

$\sqrt{s}$ (TeV)	$L_\perp$ (fm)	λ (GeV)	w	N	$\langle p_T \rangle$ (GeV)	dN/dy
0.2	0.726	0.269	2.76	33.6	0.38	4.0
1.0	0.745	0.308	3.96	59.2	0.43	5.5
2.0	0.753	0.331	4.70	75.8	0.46	6.5
7.0	0.792	0.383	7.03	126.6	0.54	9.6

We assume spatial transverse homogeneity and a collision area of size L^2. Output is the total multiplicity N ,the mean transverse momentum and the rapidity distribution at y=0. We use parton/hadron duality to convert the gluon distribution into a pion distribution.

Multiplicity distributions:

$$\frac{dN}{dy d^2p_\perp} = \frac{gL_\perp^2}{(2\pi)^2} \frac{1}{\exp\left[m_\perp\left(\frac{1}{\lambda} + \frac{we^{|y|}}{\sqrt{s}}\right)\right] - 1}$$

$$\frac{dN}{dy} \approx \frac{\pi g L_\perp^2 \lambda^2}{12\left(1 + w\lambda\frac{e^{|y|}}{\sqrt{s}}\right)^2}.$$

Parton-hadron duality pre-scription is to replace transverse momentum by transverse mass

$$m_\perp = \sqrt{m_\pi^2 + p_\perp^2}.$$

Corrections to the above formula are of order pion mass/effective transverse temperature

RHIC pp-Data at 200 GeV

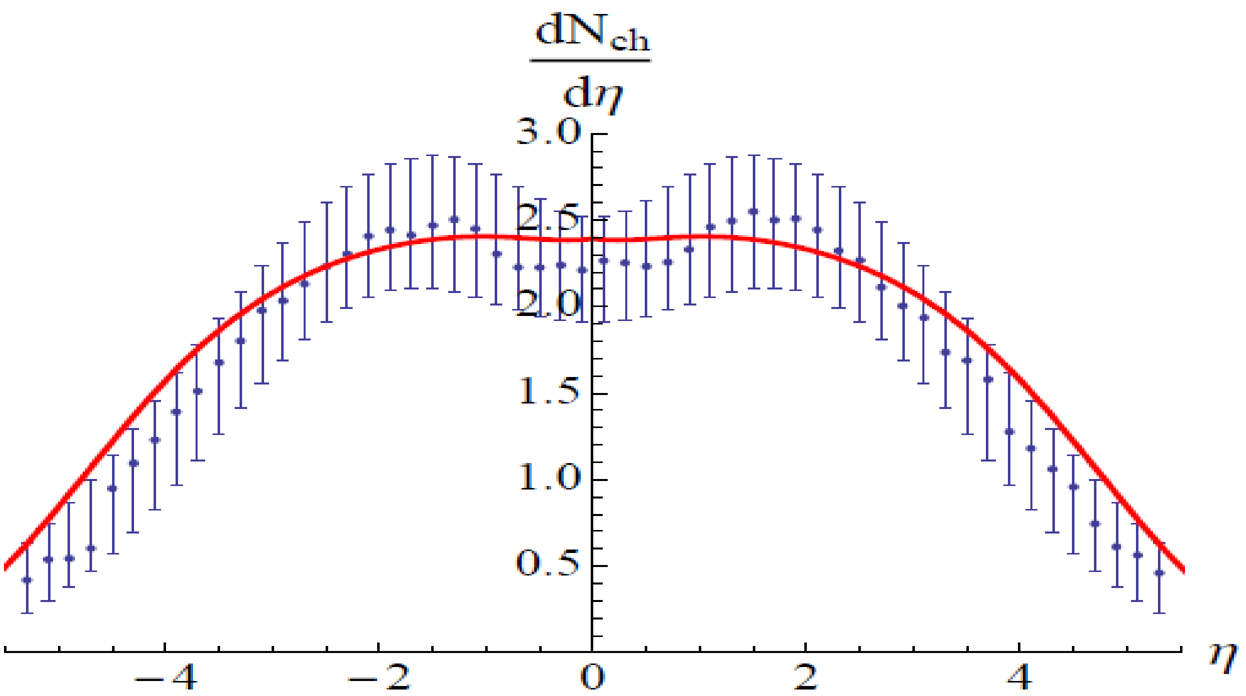


Figure 1: Data points show the charged-particle pseudo-rapidity distribution in p+p collisions at $\sqrt{s} = 200\,\mathrm{GeV}$ from [11], the full drawn curve represents the result following from the light-cone plasma distribution.

LHC data at 7000 GeV

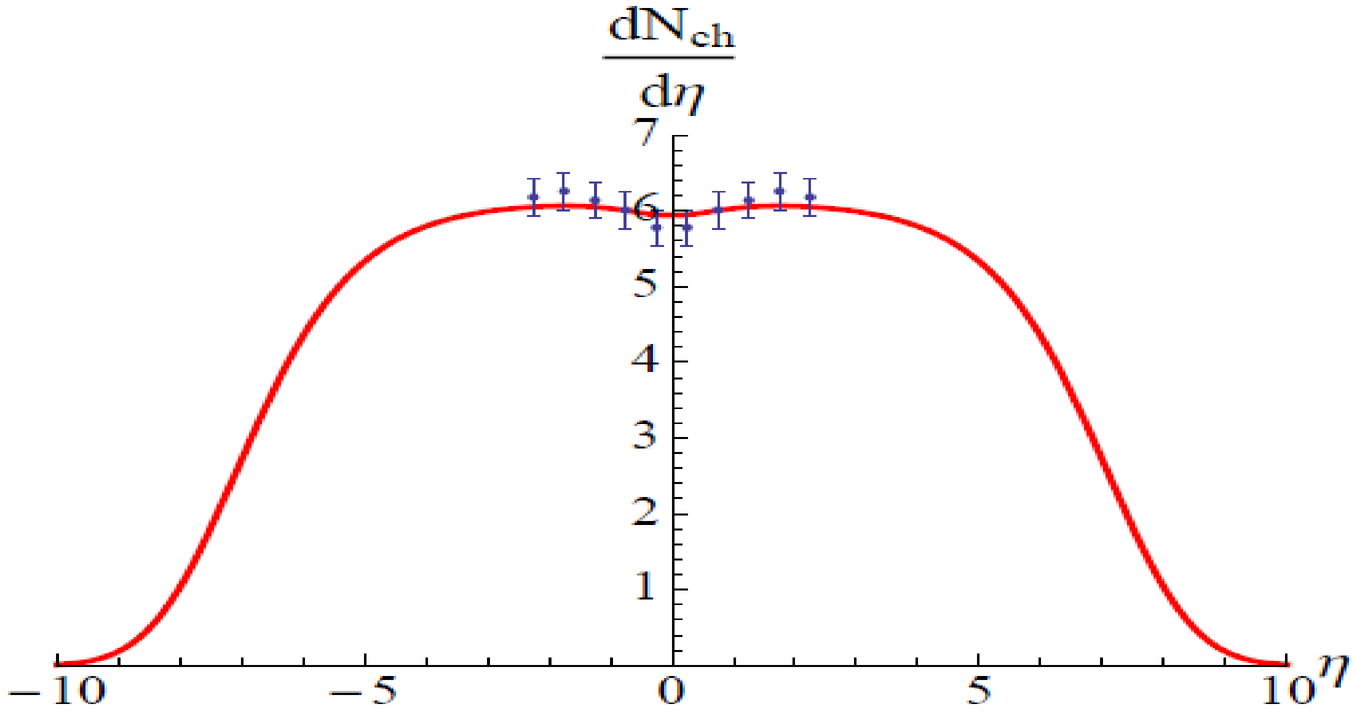


Figure 2: Data points show the charged-particle pseudorapidity distribution in p+p collisions at $\sqrt{s}$ = 7000 GeV from [7], the full drawn curve represents the result from the light-cone plasma distribution.

How to generalize this to AA-collisions?

- Since we are dealing with soft collisions we tried to rescale the pp-distribution with the number of participants- this is not sufficient
- We need in addition the experimental fact that the mean transverse momentum of the partons, released by the wounded nucleons is broadened in the nucleus-nucleus collisions

Multiplicity distribution in AA-collision

$$\frac{dN_{ch}^{AA}}{d\eta d^2 p_\perp} = \frac{N_{\text{part}}}{2}\frac{2}{3}\sqrt{1 - \frac{m_\pi^2}{m_\perp^2 \cosh^2 y}}\frac{dN(\langle p_\perp \rangle)}{dy d^2 p_\perp}.$$

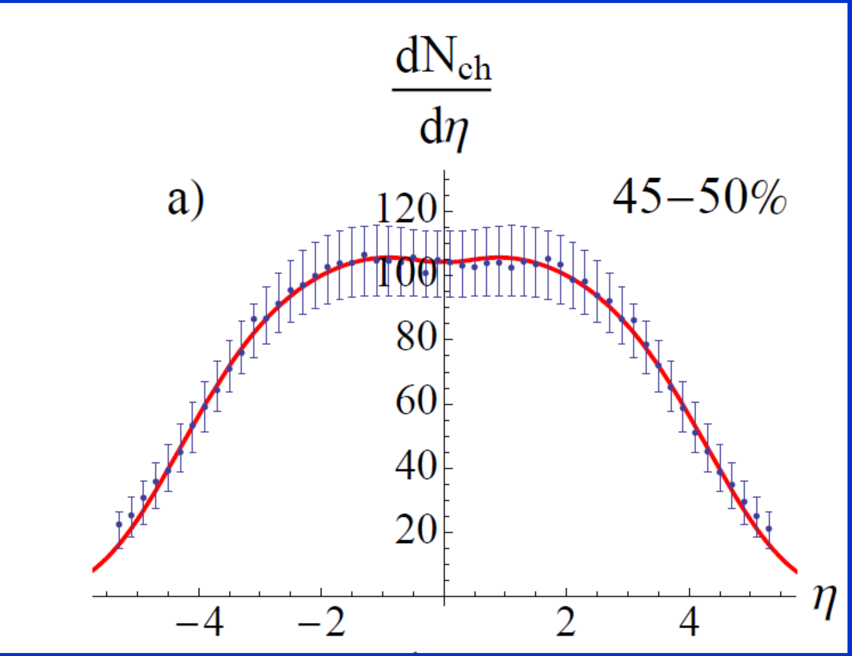
The pp-distribution in each hemisphere gets multiplied with the number of participants and its transverse temperature gets rescaled to reproduce the measured mean transverse momentum

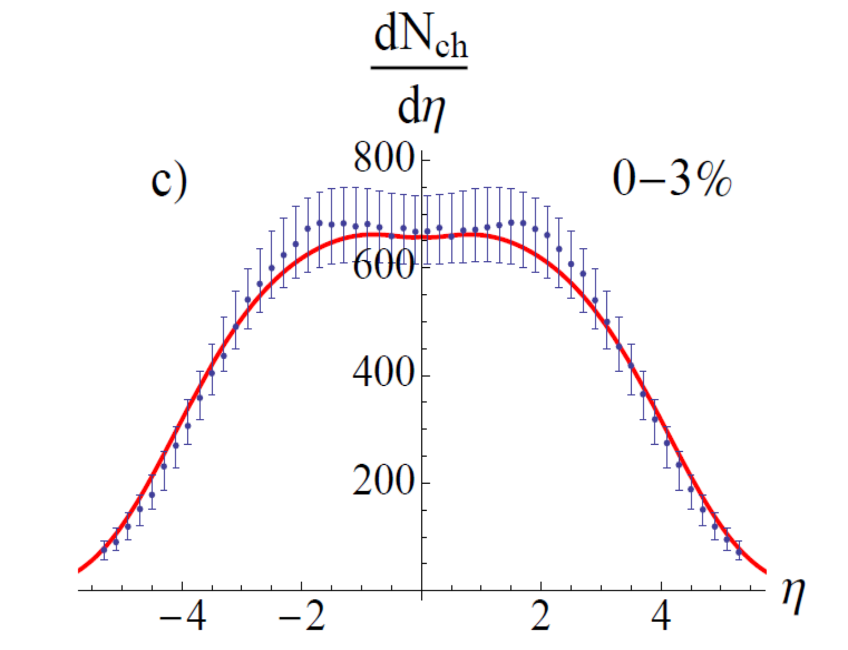
Light Cone distribution for Au-Au Collisions at different centralities

centr.	N_{part}	λ (GeV)	w	N	$\langle p_T \rangle$ (GeV)	$dN_{ch}^{AA}/d\eta$
45-50%	65	0.307	3.71	1297	0.423	104
25-30%	150	0.323	4.13	3192	0.443	267
0-3 %	361	0.327	4.24	7803	0.448	658

Input parameters are Npart, λ from experiment , the w- is obtained from the constraint, then N, mean transverse momentum und dN/dη follow

Pseudorapidity distributions as function of centrality





The increase of dN/dη follows the effective transverse temperature

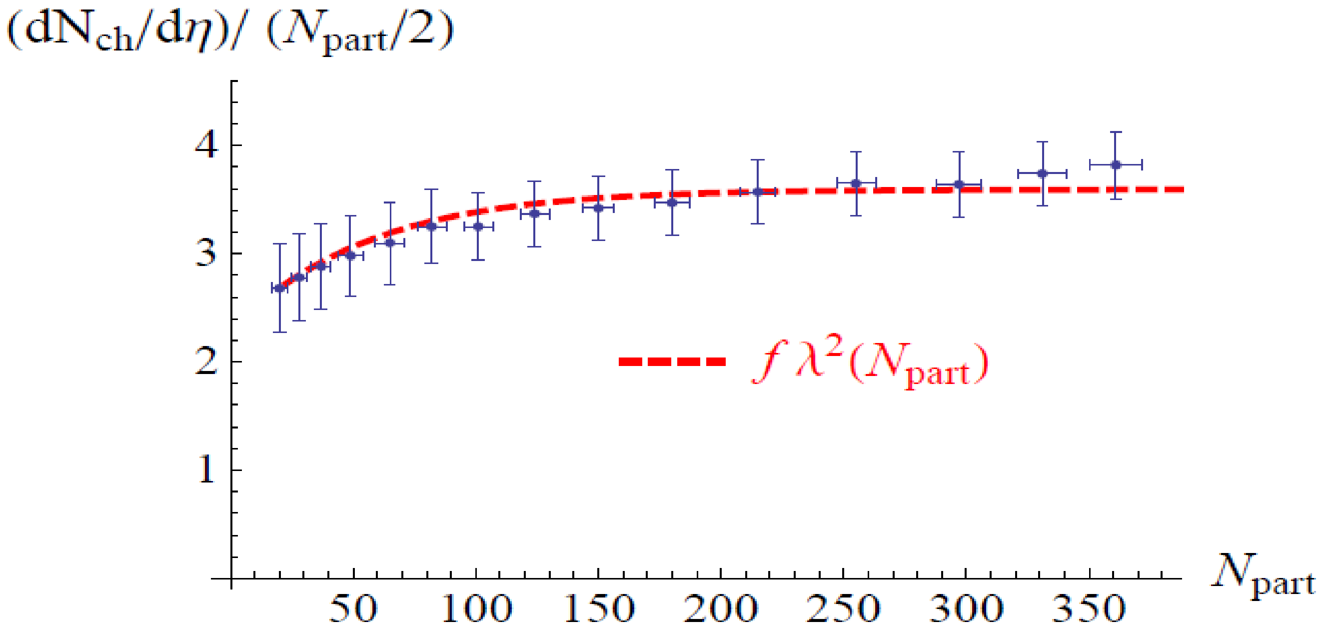


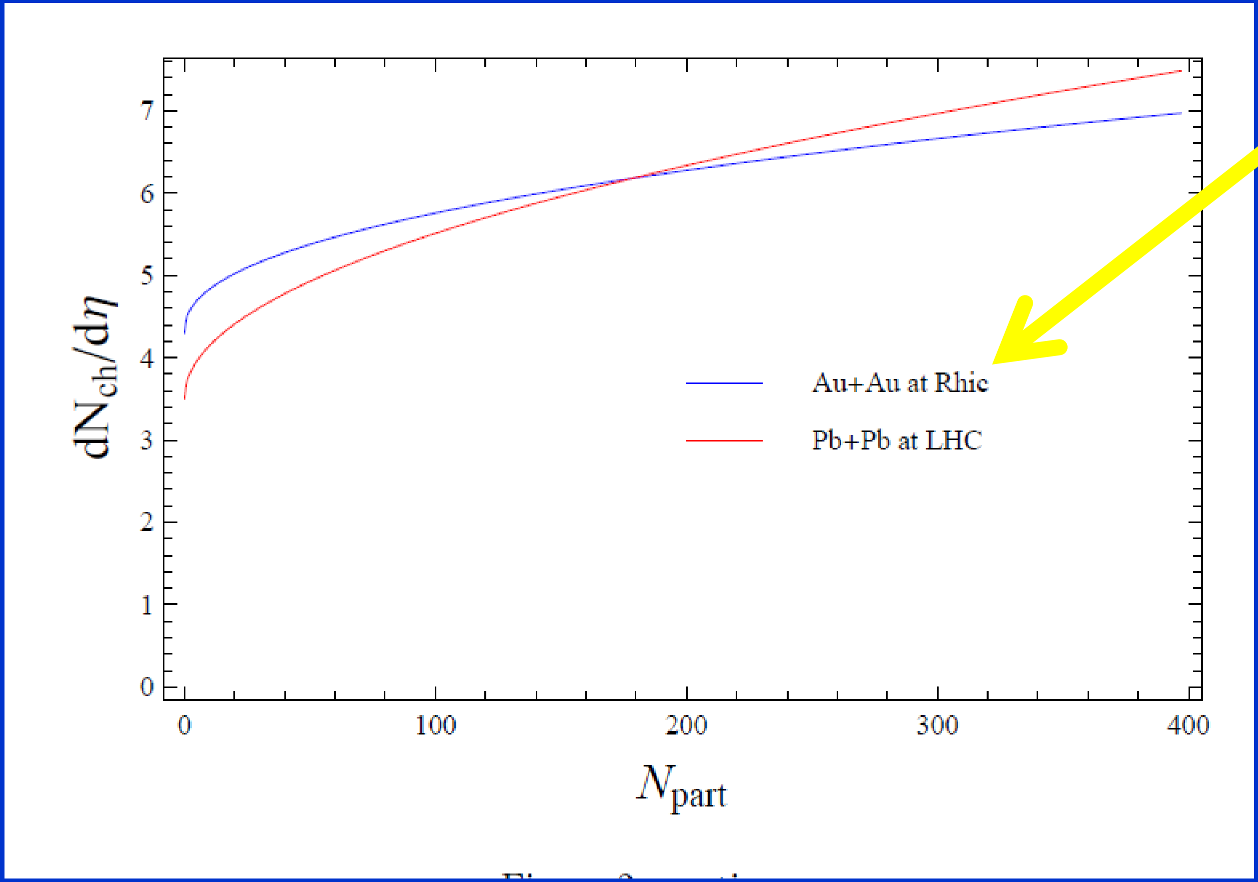
Figure 4: Square of the effective transverse temperature $\lambda^2(N_{\rm part})$ compared with Phobos data from [11] for $(dN_{ch}/d\eta)/(N_{\rm part})/2$ in Au+Au collisions at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$. The factor f was chosen as $f = (dN_{ch}/d\eta)/(N_{\rm part})/2\big|_{N_{\rm part}=215} / \lambda^2(215)$.

Microscopically we can calculate the increase of λ with centrality from multiple scattering:

$$< \Delta p_\perp^2 > = << \sigma p_\perp^2 >_g >< T_B(b) >< z^2 >_{\pi/g}$$

Transverse momentum broadening of the pion results from the mean transverse momentum of a gluon times the dipole nucleon cross section , when it traverses a path T(b)=ρ L in the other cold nucleus,
<z^2> converts the mean gluon transverse momentum to the pion transverse momentum (no longer parton/hadron duality)

dN/dη /(Npart/2) distributions are given by the broadened light cone distribution



Times 2

Mutual boosting of saturation scales:

$$Q_s^2 = < \Delta p_\perp^2 >$$

Transverse momentum broadening defines the saturation scale

$$\frac{2}{9}\overline{< \sigma p_\perp^2 >_g} = \frac{\pi^2}{3}\alpha_s(Q_0^2)\overline{\bar{x} g_N(\bar{x}, Q_0^2)},$$

$$\bar{x} = \frac{Q_0^2}{\hat{s}} = \frac{Q_0^2}{sx}.$$

The starting scale has to make the two gluon exchange picture coincide with the average transport parameter

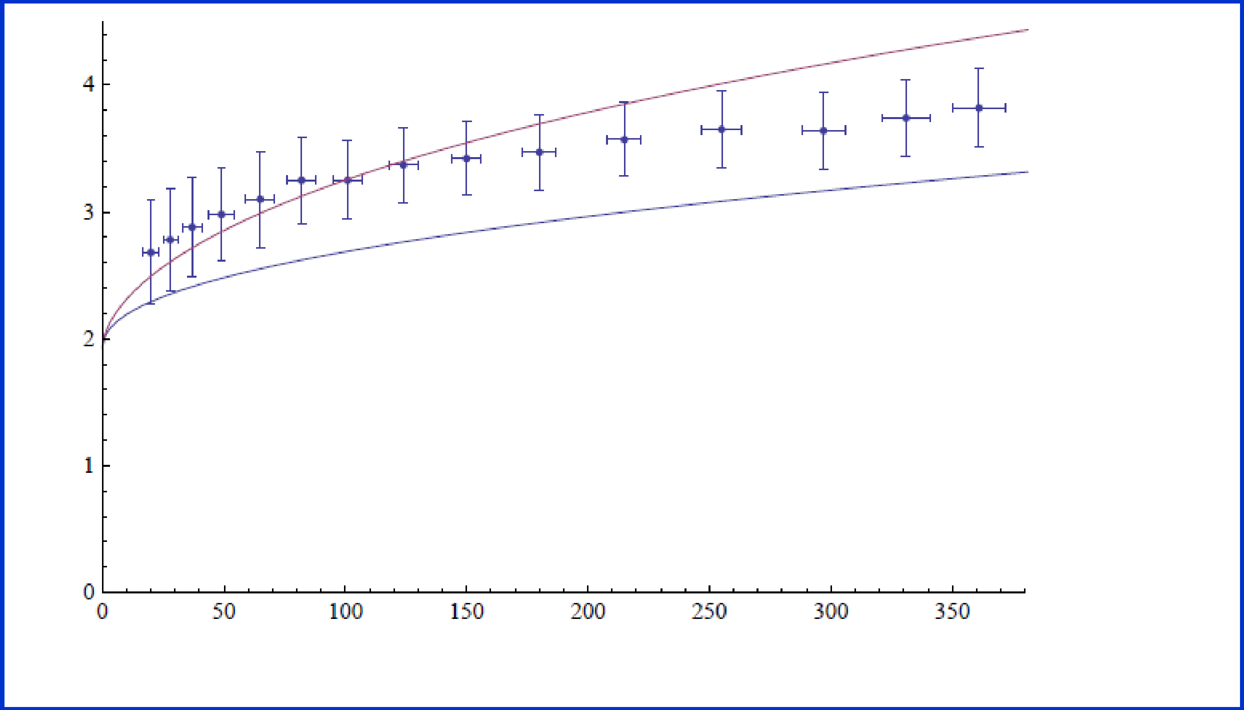
Mutual boosting

$$\frac{dQ^2_{sB}}{d < T_B(b) >} = \frac{3\pi^2}{2}\alpha_s(Q^2_{sA} + Q^2_0)\overline{\bar{x}_A g_N(\bar{x}_A, Q^2_{sA} + Q^2_0)}$$

$$\frac{dQ^2_{sA}}{d < T_A(b) >} = \frac{3\pi^2}{2}\alpha_s(Q^2_{sB} + Q^2_0)\overline{\bar{x}_B g_N(\bar{x}_B, Q^2_{sB} + Q^2_0)}$$

Here, the increase of the saturation scale dQ^2_{sB} in nucleus B is given by the $p_\perp$-broadening of a parton of nucleus A going through nucleus B for an infinitesimal length $d < T_B(b) >$. Since the saturation scale Q^2_{sB} of nucleus B increases with the saturation scale Q^2_{sA} of nucleus A, Kopeliovich et al. [] have called this phenomenon mutual boosting. With increasing saturation scale also the associated Bjoken variables $\bar{x}_A$ and $\bar{x}_B$ in the gluon structure functions change:

dN/deta /(Npart/2) with and without mutual boosting as a function of number of participants



Preliminary Result !

Conclusions:

- There is a universal maximum entropy distribution satisfying the constraints of a given transverse energy and unity-sum of parton light cone momentum fractions
- It is nonthermal and has as parameters: transverse temperature λ and softness w
- Direct calculation of the entropy and comparison with boosted thermal shows that light cone entropy is bigger (% effect)

Conclusions:

- For gluon dominance, it gives a good fit to the pp-data at RHIC and LHC using a reasonable overlap area $L^2=(0.7\ \mathrm{fm})^2$ and the correct number of degrees of freedom
- Using participant scaling and transverse momentum broadening it also explains the AA-multiplicity distributions as functions of centrality
- There may be a nonadditive effect of mutual boosting of saturation scales in AA-collisions