Spin transport in 2D Fermi gases

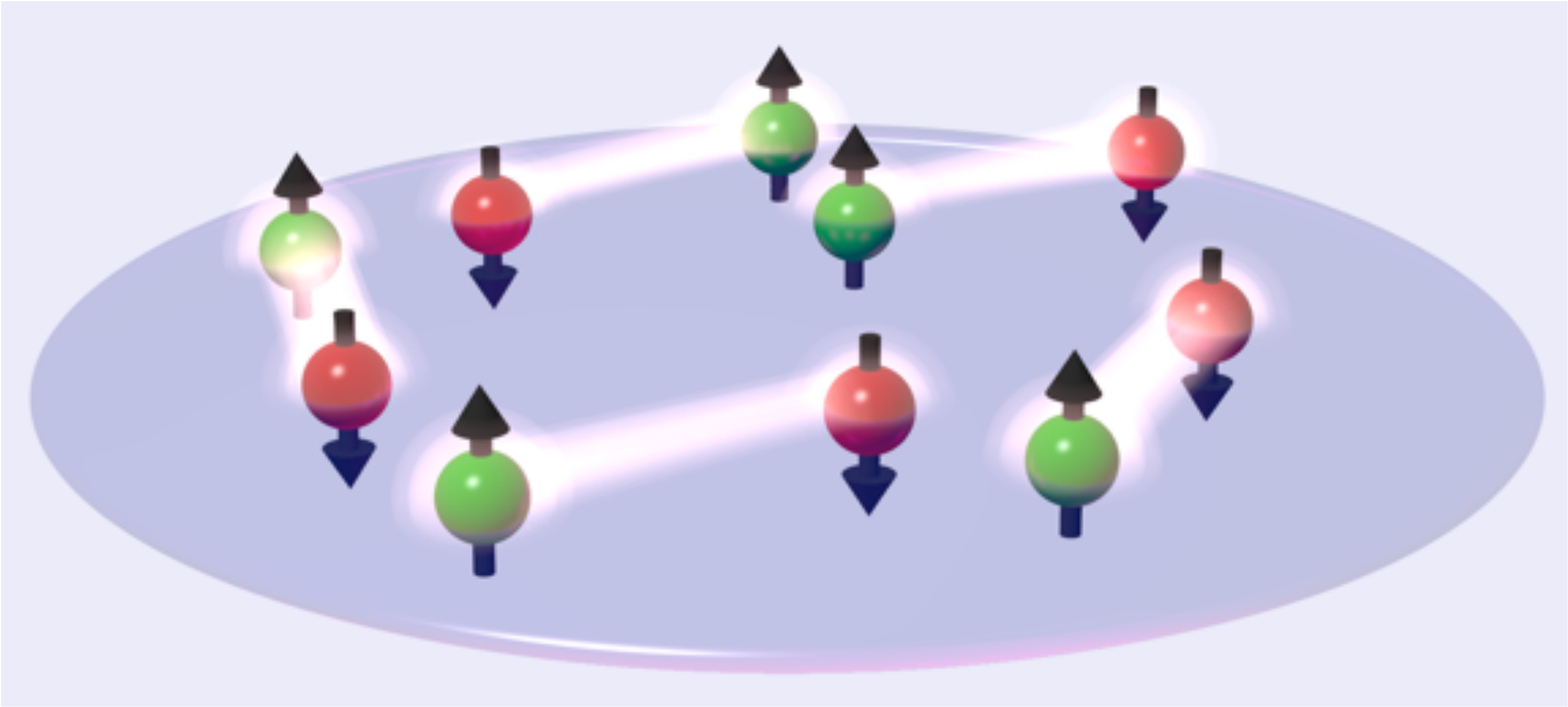
dimensionality, scale invariance and strong interaction

Tilman Enss (University of Heidelberg)

Nicolò Defenu (Heidelberg, theory)
Jochim group (Heidelberg, expt)
Thywissen group (Toronto, expt)

Frontiers in 2D Quantum Systems
Trieste, 14 Nov 2017

2D Fermi gas



dilute gas of ⬆ and ⬇ fermions with contact interaction:

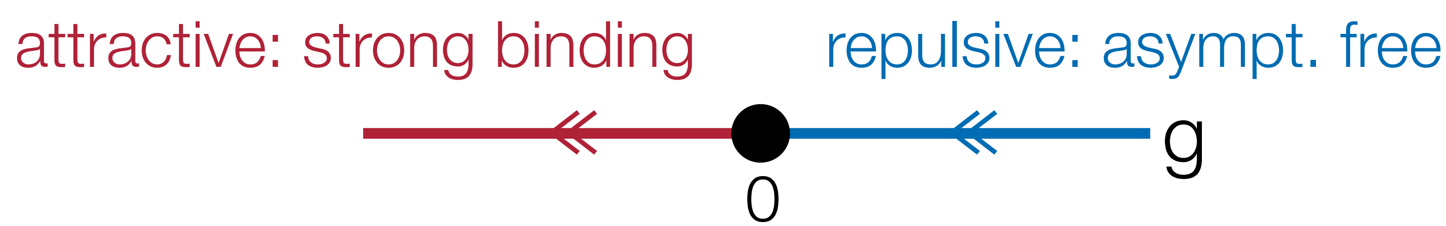
$$\mathcal{H} = \int d\mathbf{x} \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big(-\frac{\hbar^2\nabla^2}{2m} - \mu_\sigma\Big)\psi_\sigma + g_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

Levinsen & Parish Annu. Rev. CMP 2015

Scattering properties

- two-particle scattering:
  how does coupling g change when zooming out?

$$\frac{dg}{d\ln k} = \frac{g^2}{2}$$



- coupling always energy dependent (**log. running coupling**)
- **never scale invariant** (quantum anomaly breaks classical scale invariance)

Holstein 1993; Pitaevskii & Rosch 1997

exact two-body scattering amplitude:

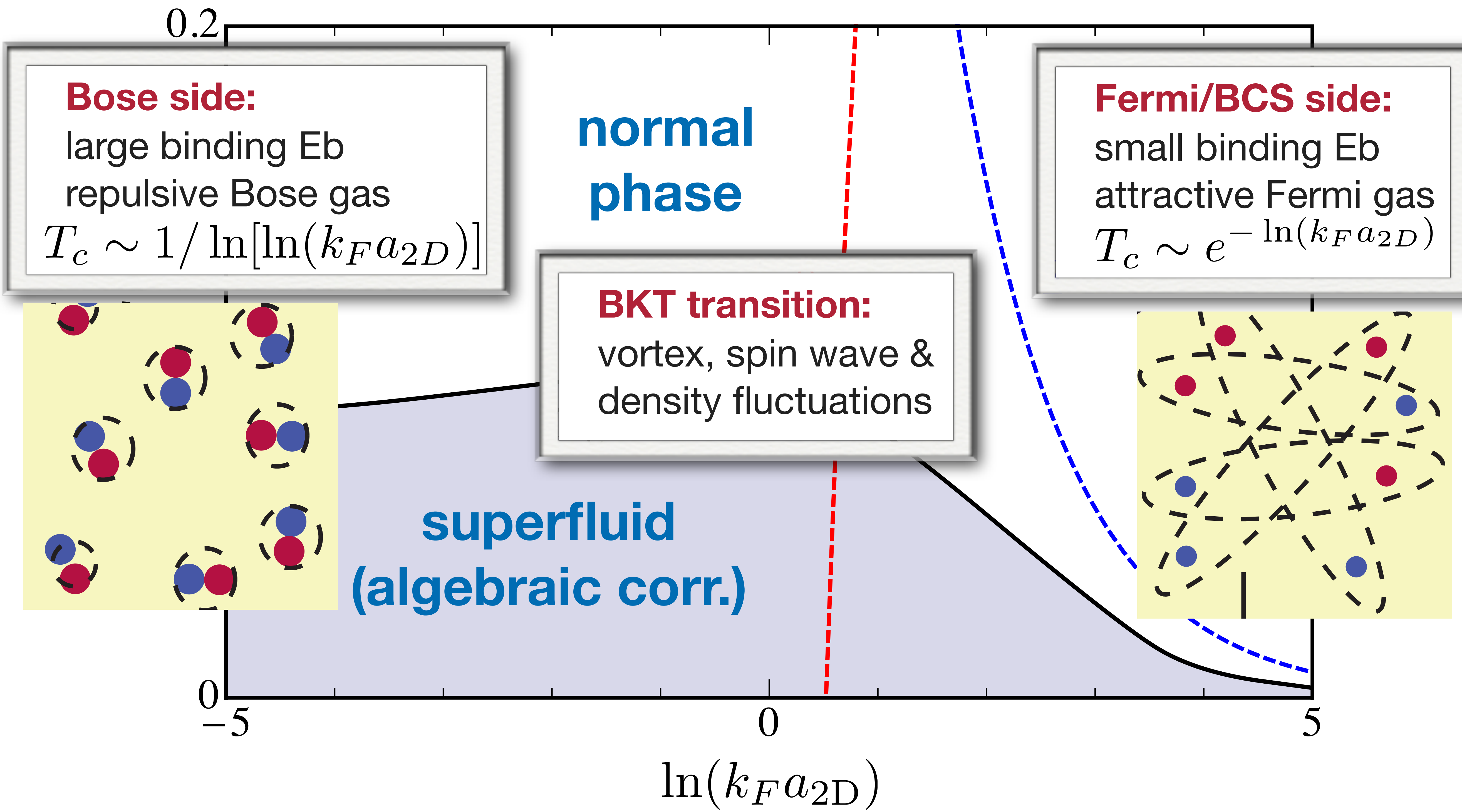
$$f(k) = \frac{4\pi}{\ln(-\varepsilon_B/E_k)} = \frac{2\pi}{i\frac{\pi}{2} - \ln(ka_{2D})}$$

always bound state

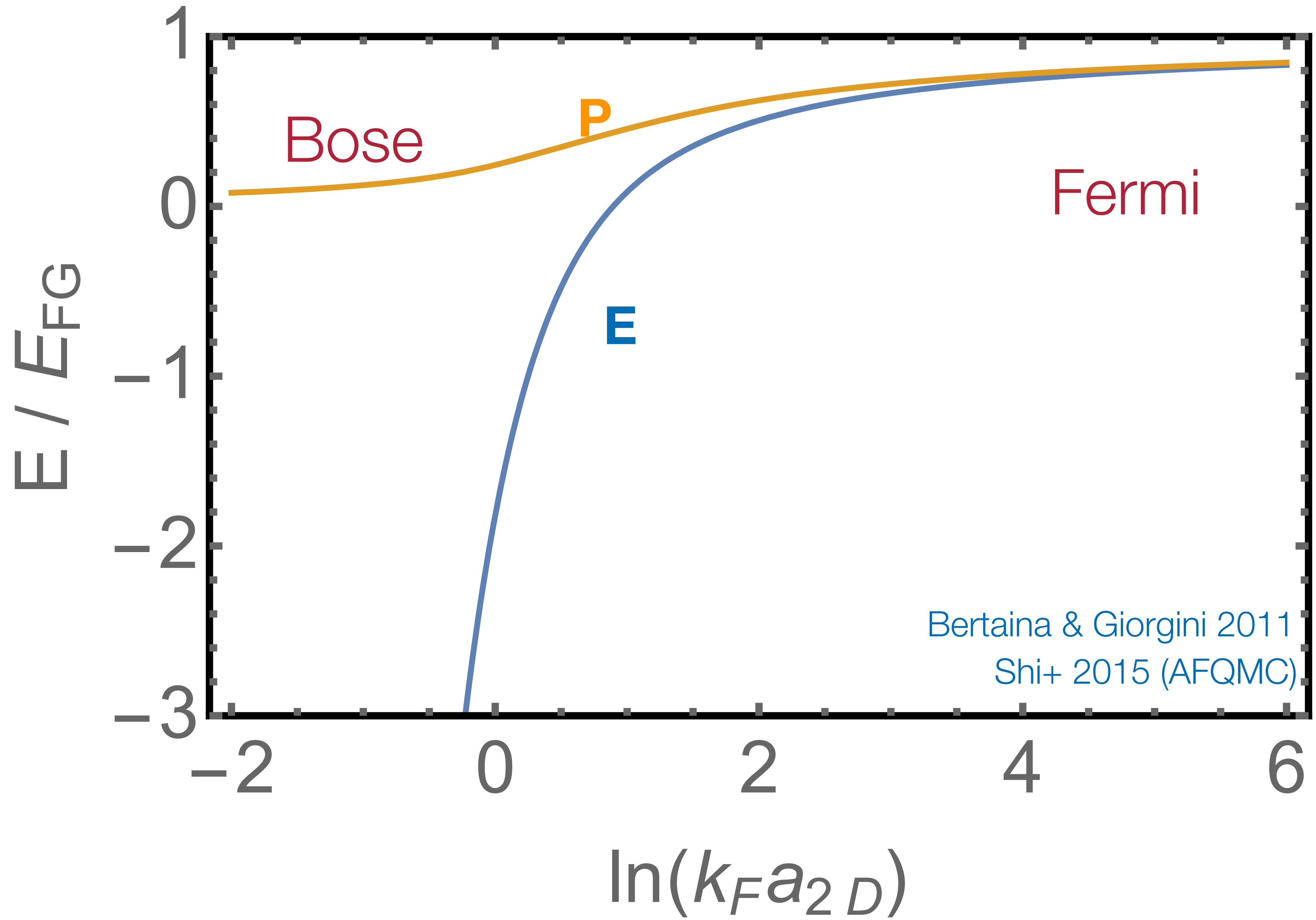
$$\varepsilon_B = \frac{\hbar^2}{m a_{\mathrm{2D}}^2}$$

- typical scale $k=k_F$: interaction parameter $\ln(k_F a_{2D})$

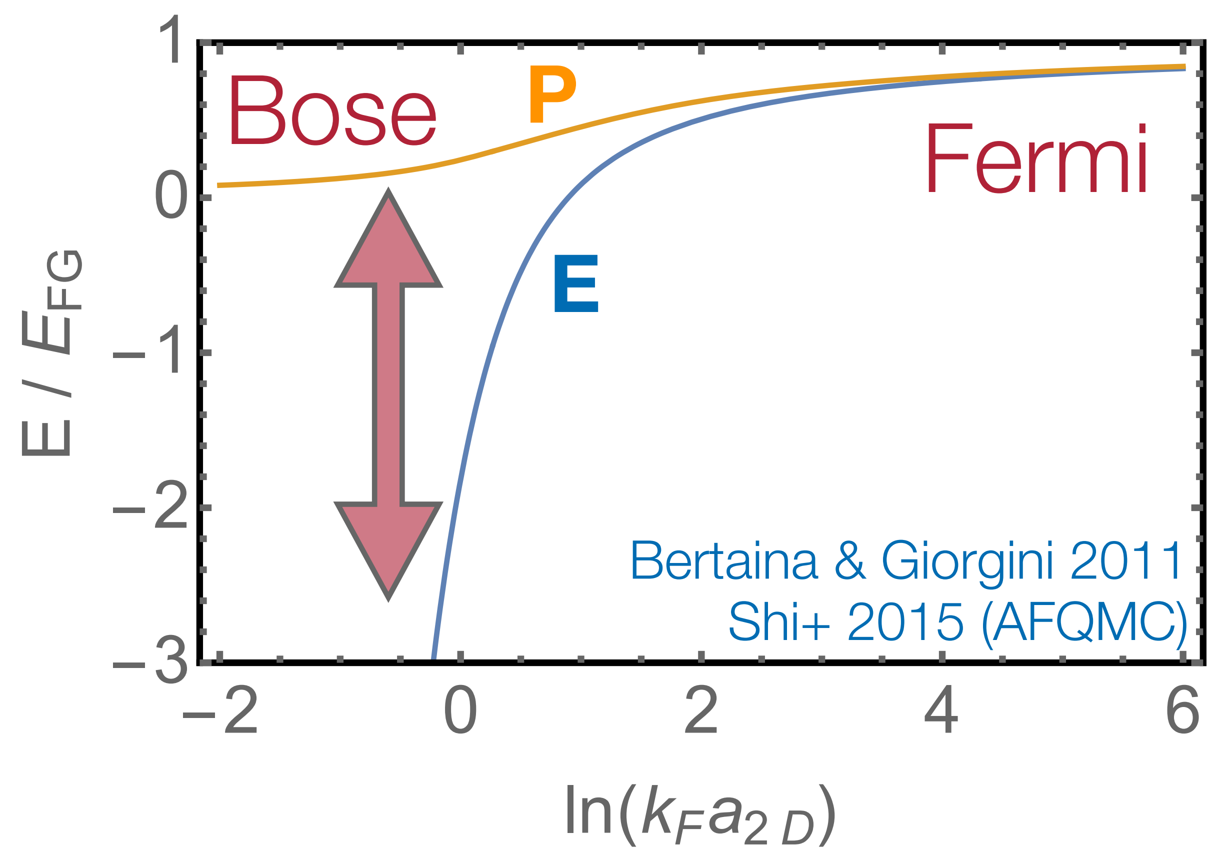
Phase diagram



Thermodynamics & scale invariance



Thermodynamics & scale invariance



scale invariance in 2D:

$$P = E$$

interacting 2D Fermi gas:

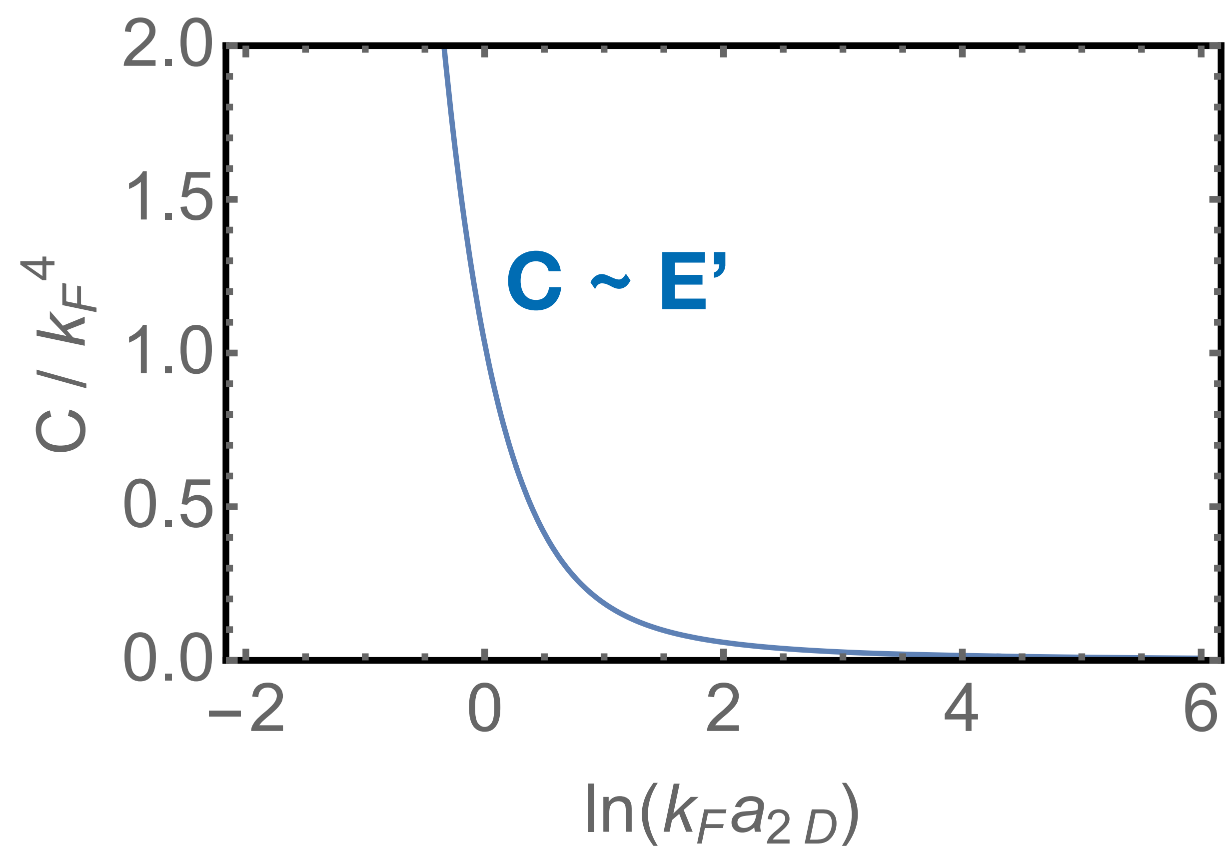
$$P = E + \frac{C}{4\pi m}$$

scale invariance broken

Contact density: probability to find ⬆ and ⬇ in same place

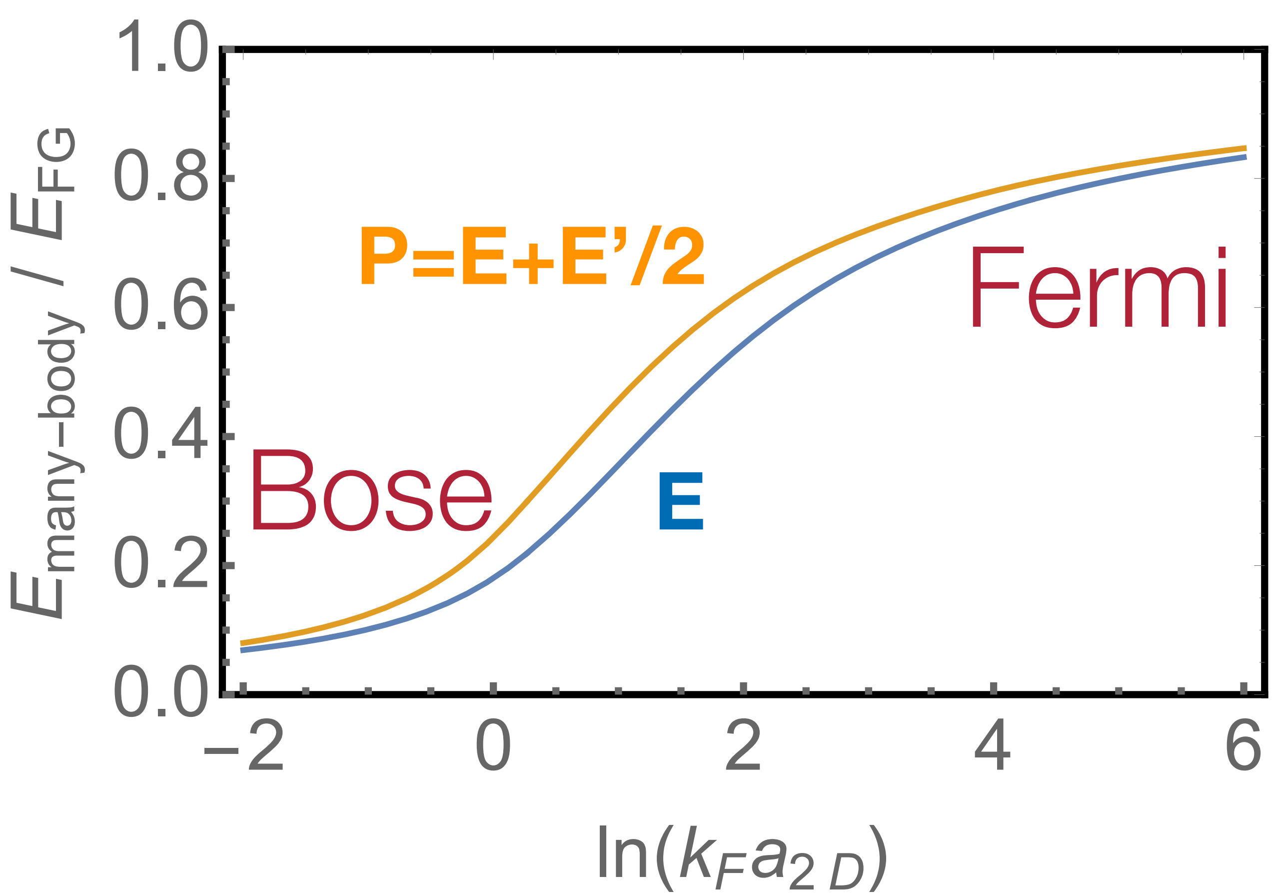
$$C = m^2 g_0^2 \left\langle \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow(r) \right\rangle$$

$$\frac{dE}{d \ln a_{2D}} = \frac{C}{2\pi m}$$

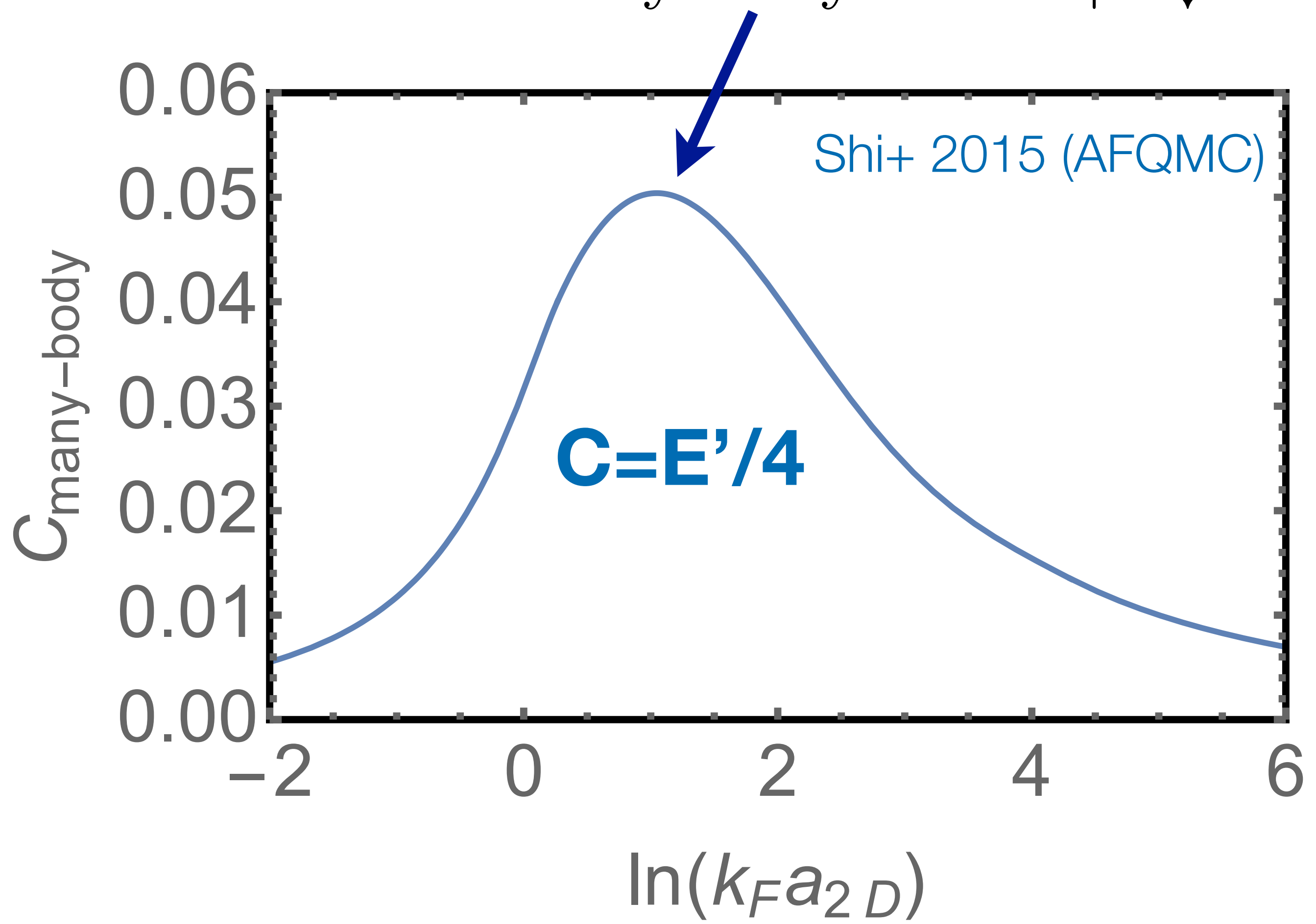


Local **many-body** correlations

subtract two-body binding energy:



$C_{\text{many-body}} \approx 8 n_\uparrow n_\downarrow$

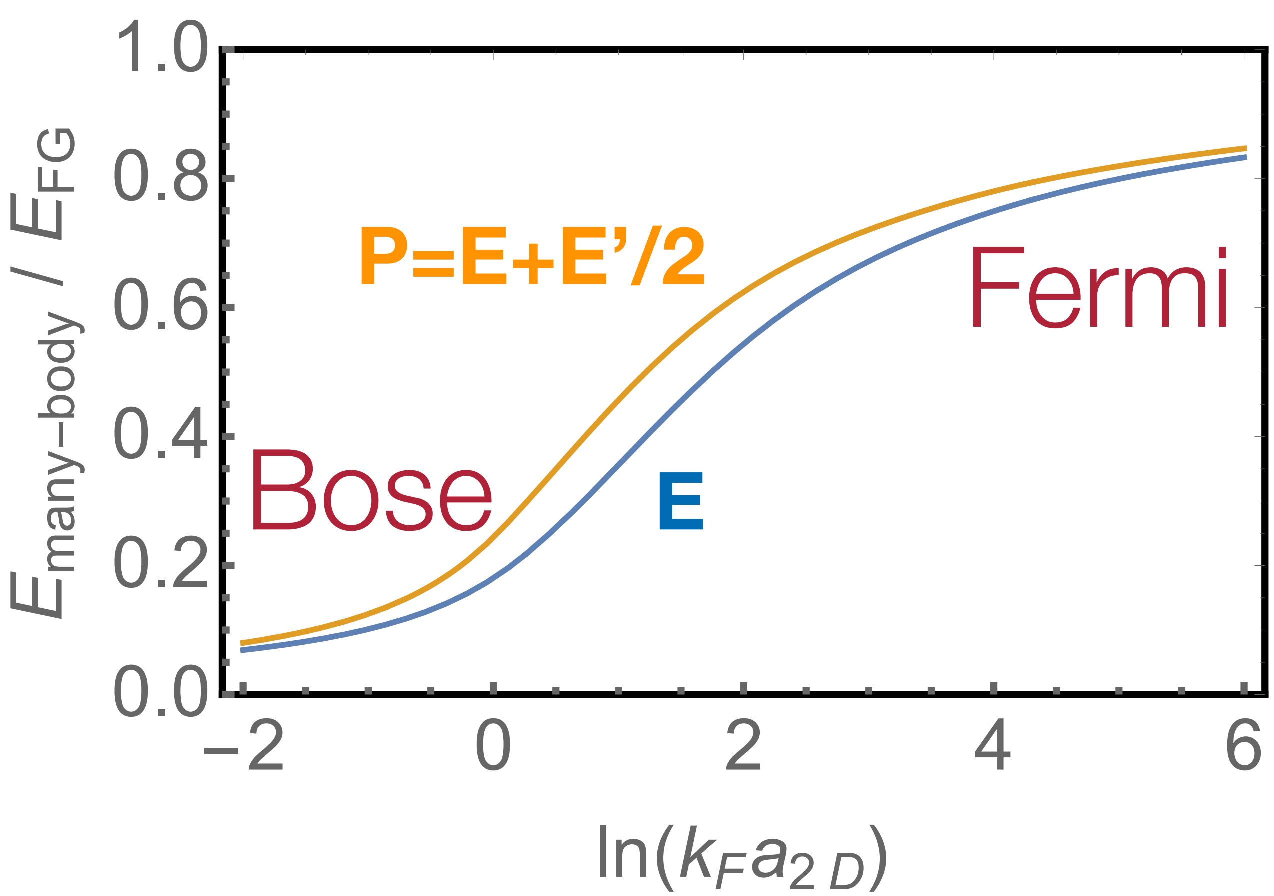


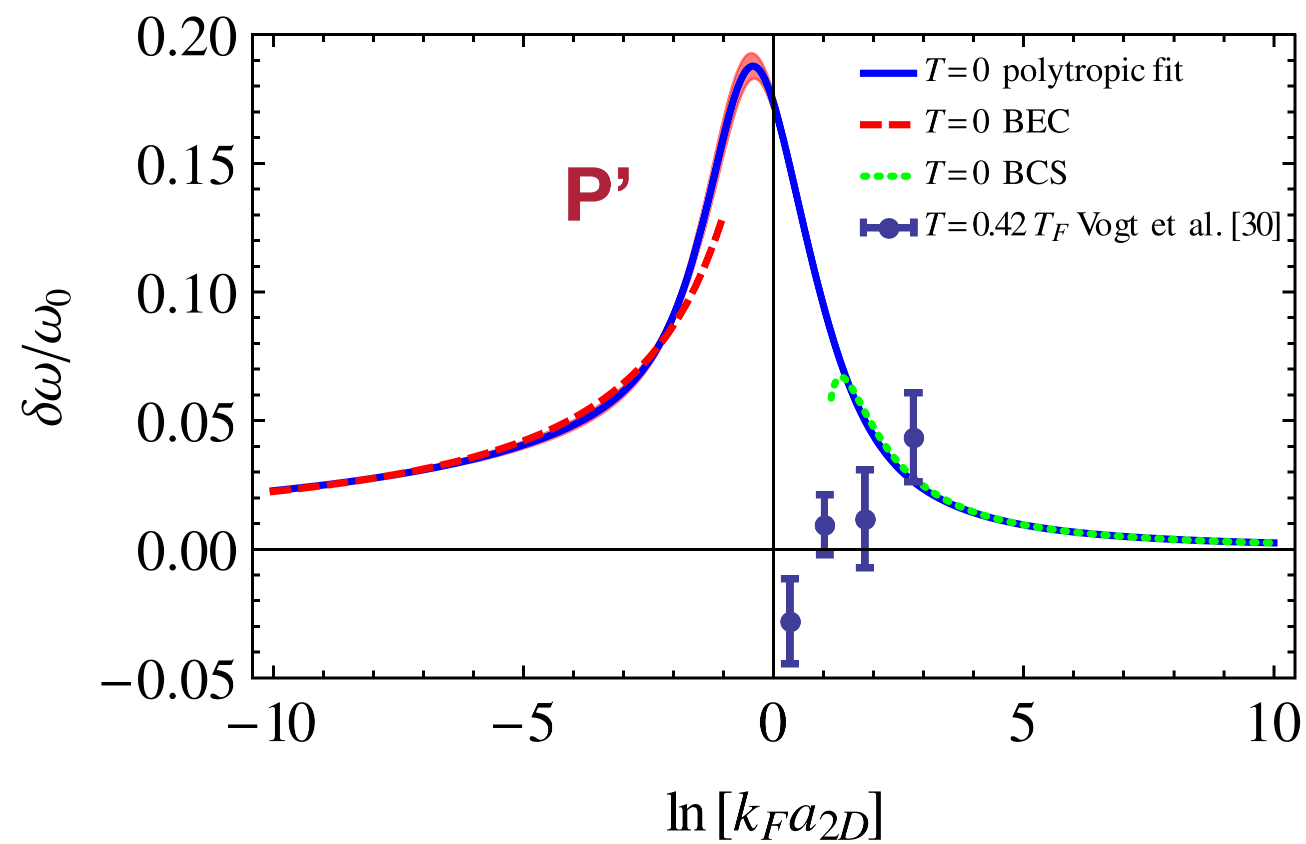
strong local correlation in crossover:
quantify scale invariance breaking

Local **many-body** correlations

subtract two-body binding energy:

breathing mode freq. = 2+P'



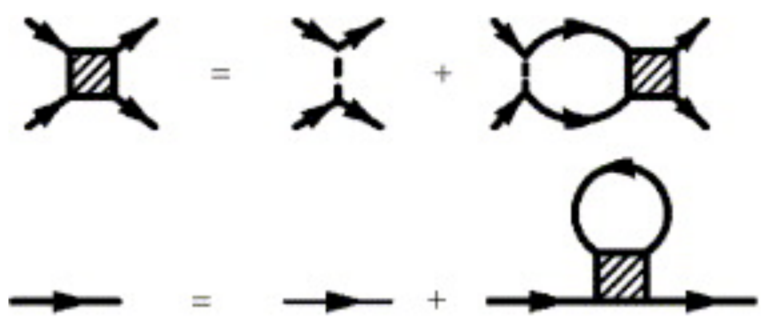


Hofmann 2012
Taylor & Randeria 2012

strong local correlation in crossover:
quantify scale invariance breaking

T>0: Luttinger-Ward approach

- repeated particle-particle scattering dominant in dilute gas:

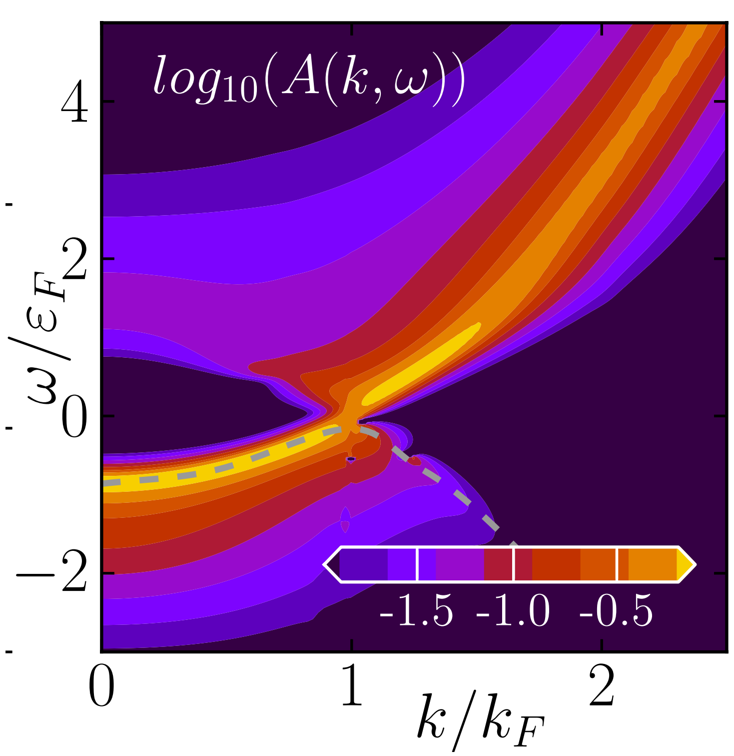


self-consistent T-matrix — Haussmann 1993, 1994; Haussmann et al. 2007

self-consistent fermion propagator
(400 momenta / 400 Matsubara frequencies)

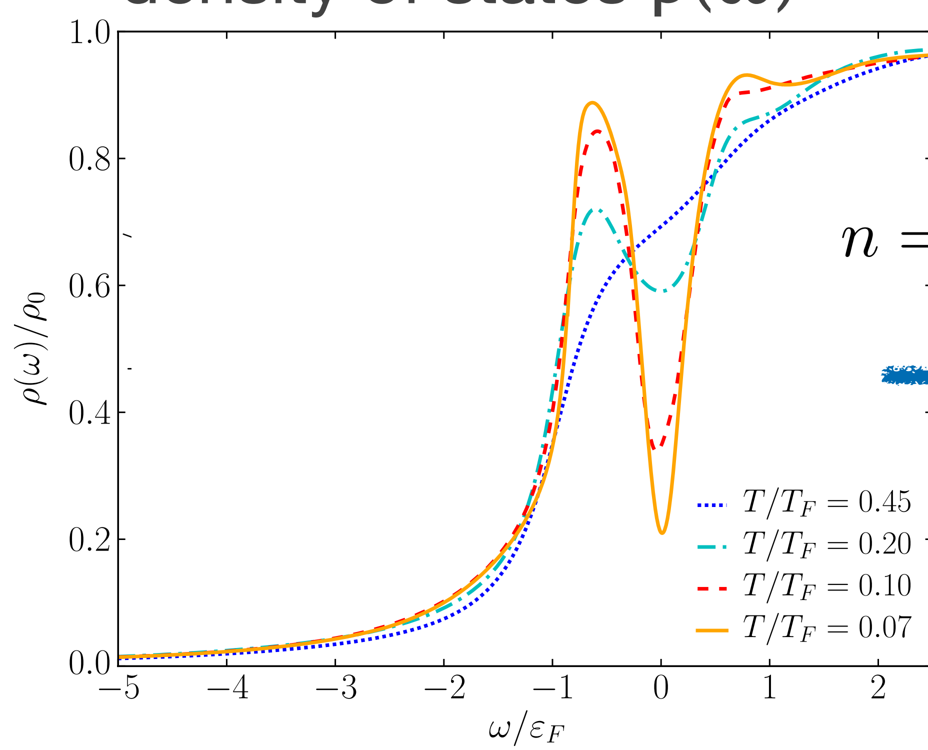
Bauer, Parish & Enss PRL 2014

- spectral function A(k,ω)



$$\int \frac{d^2k}{(2\pi)^2} A(k,\omega)$$

density of states ρ(ω)

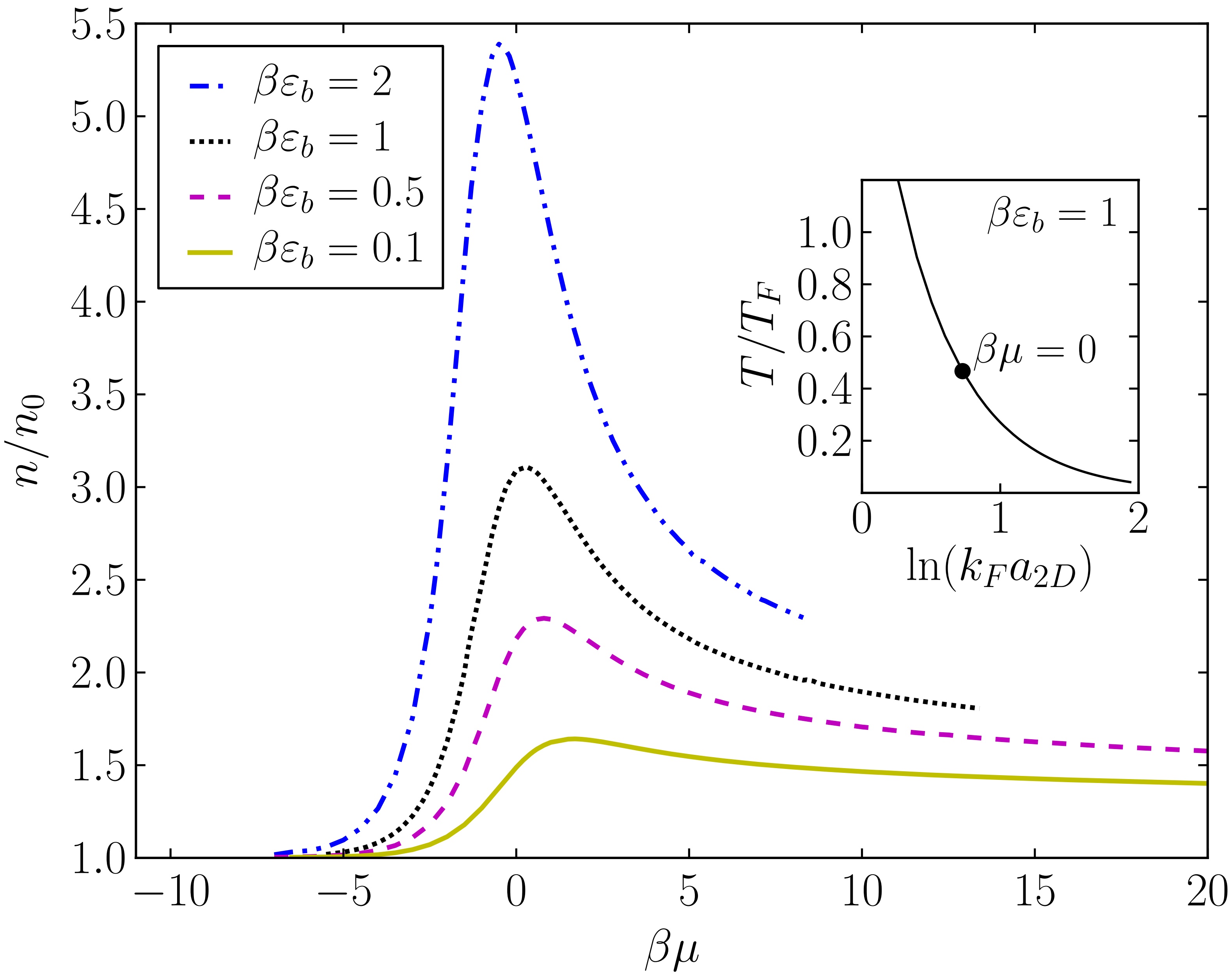


$$n = 2\int d\omega\, f(\omega)\rho(\omega)$$

density

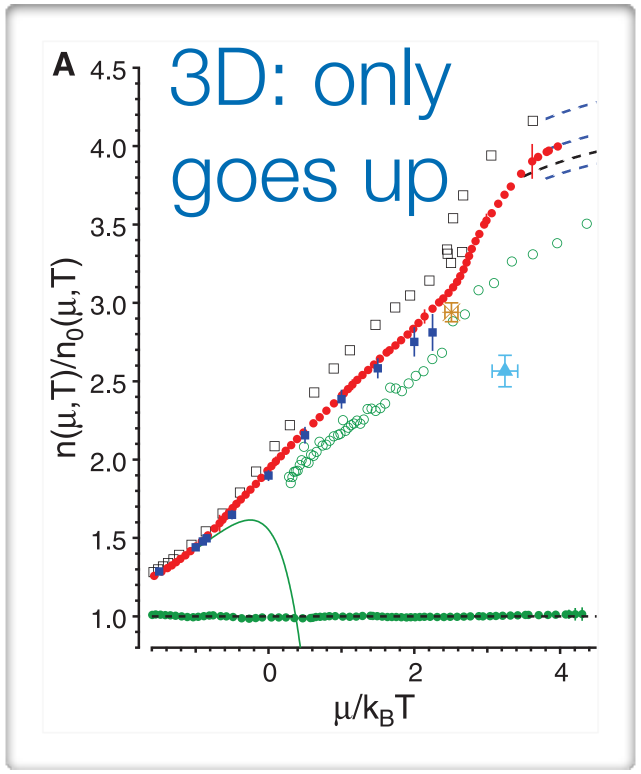
Density equation of state: theory

- **maximum** & **density driven crossover**



$$n = 2\int d\omega\, f(\omega)\rho(\omega)$$

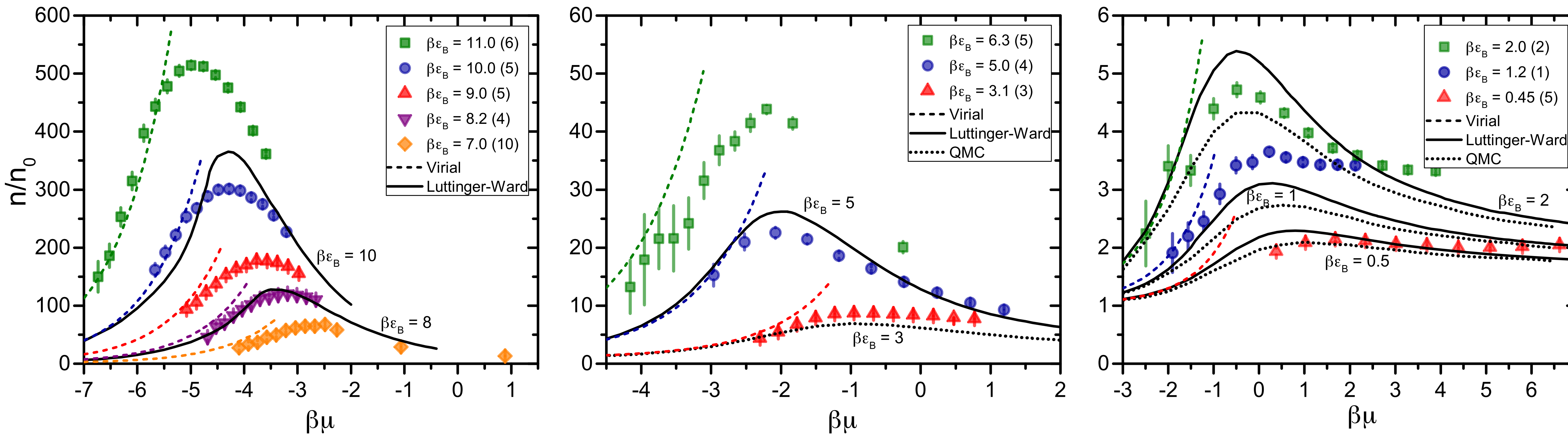
$$n_0 = 2\ln(1+e^{\beta\mu})/\lambda_T^2$$



Bauer, Parish & Enss PRL 2014

Ku+ Science 2012

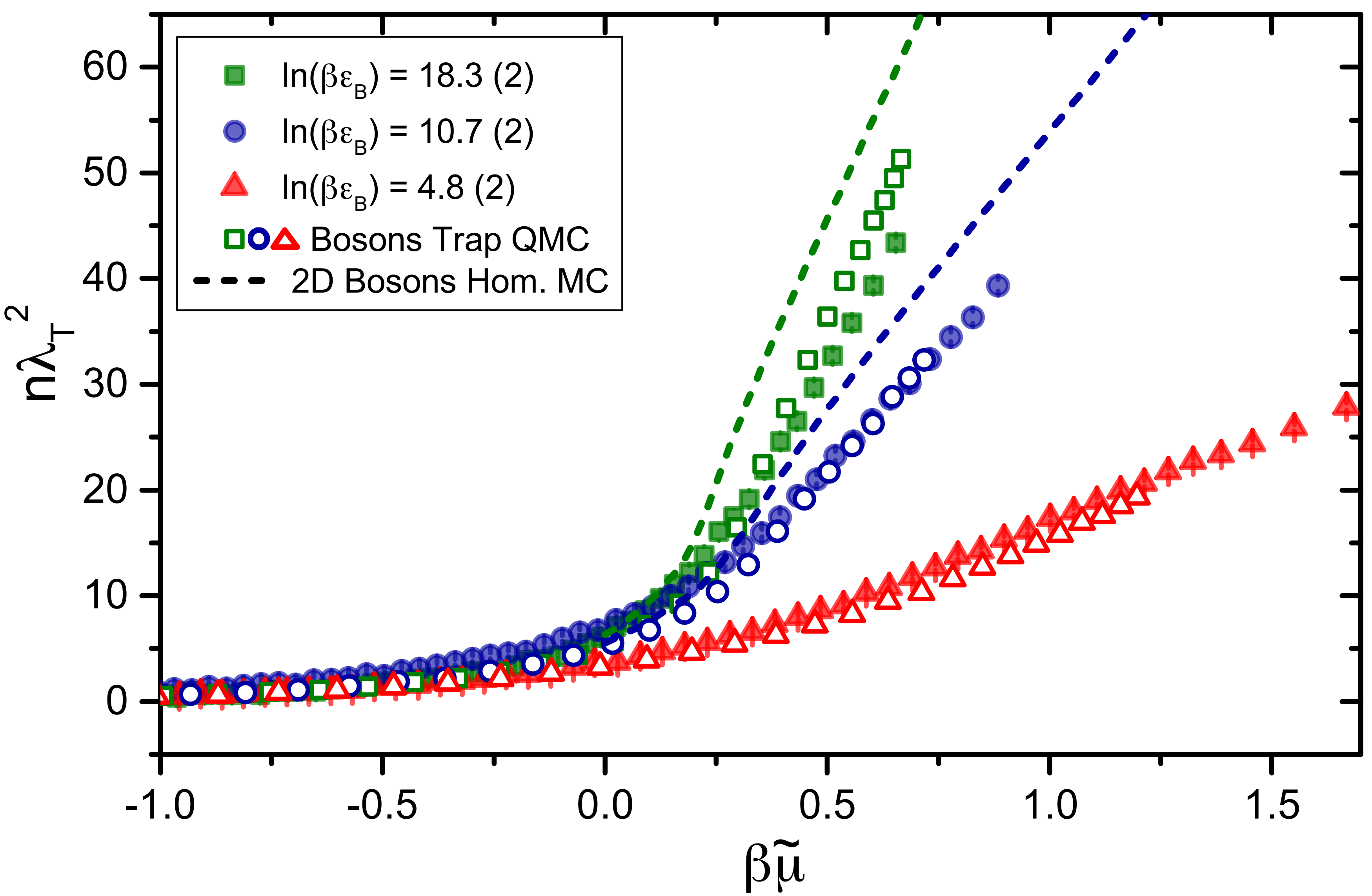
Equation of state: cold atom experiment (Jochim)



Boettcher, Bayha, Kedar, Murthy, Neidig, Ries, Wenz, Zürn, Jochim & Enss PRL 2016
see also Anderson & Drut PRL 2015 (QMC), Fenech et al. PRL 2016 (expt)

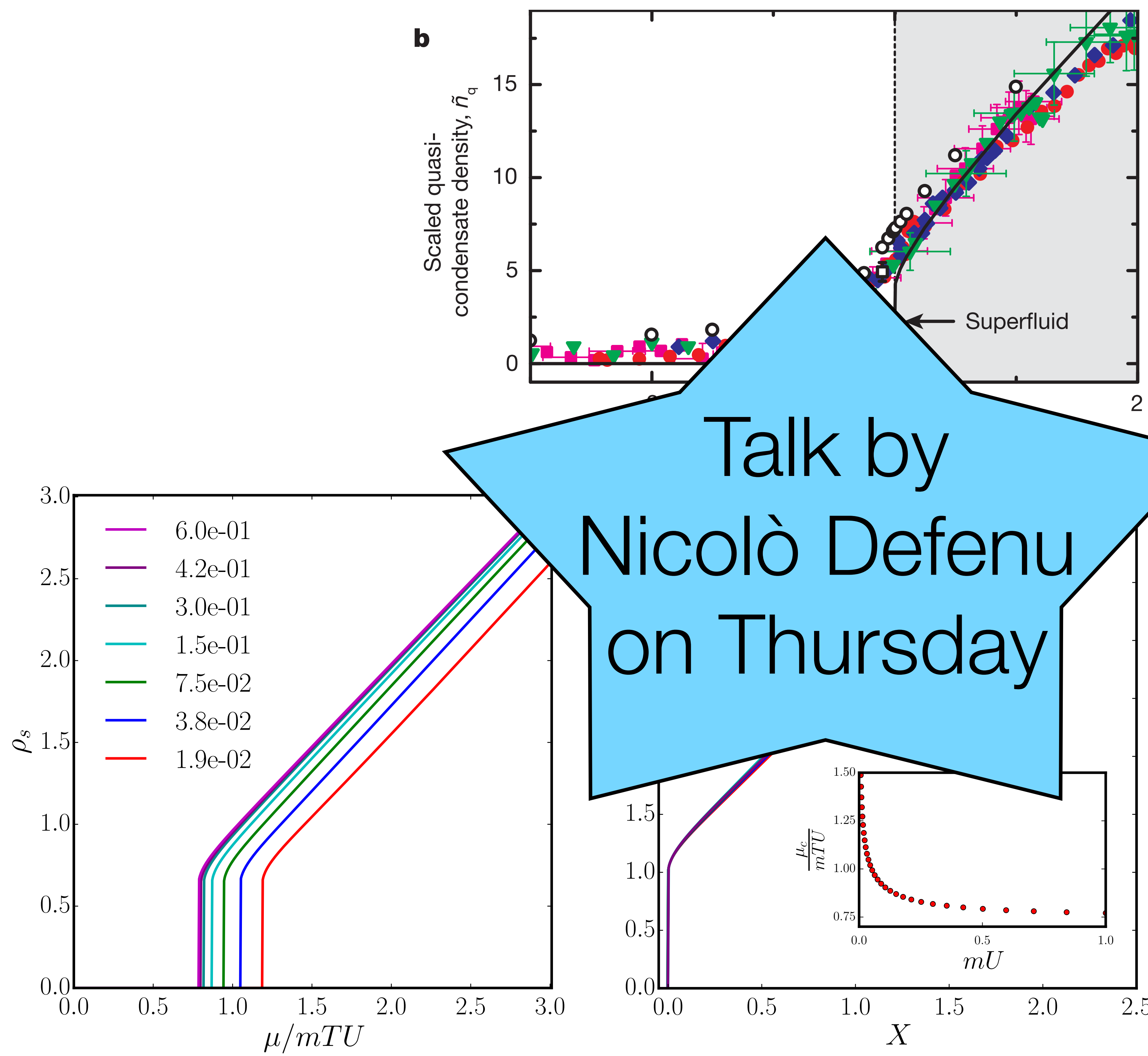
Equation of state: Bose side

- agrees with bosonic QMC in quasi-2D geometry (open symbols)



Boettcher, Bayha, Kedar, Murthy, Neidig, Ries, Wenz, Zürn, Jochim & Enss PRL 2016

Universality near critical point



Talk by
Nicolò Defenu
on Thursday

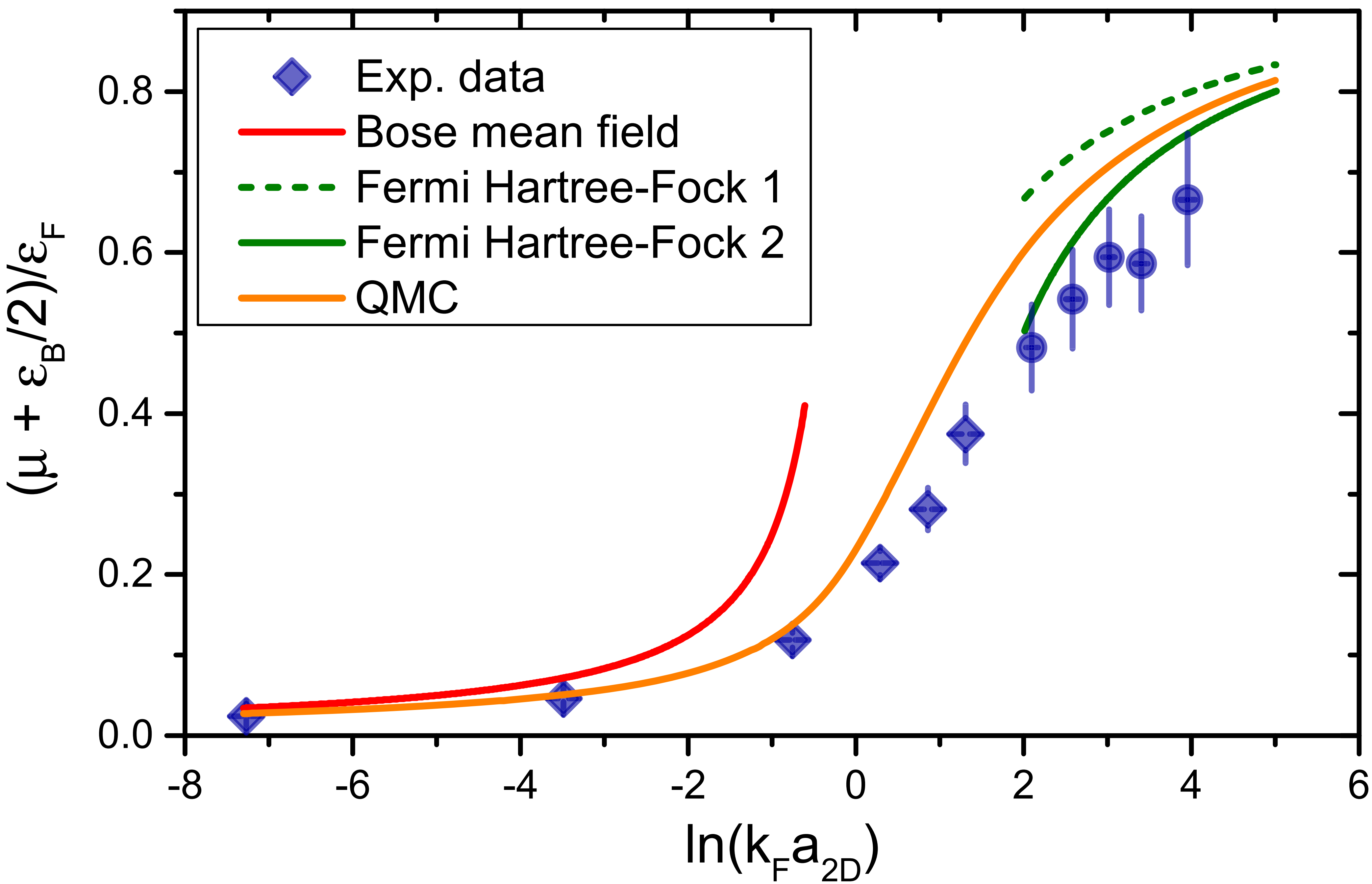
Hung+ Nature 2011

$$X = \frac{\mu - \mu_c}{mTU}$$

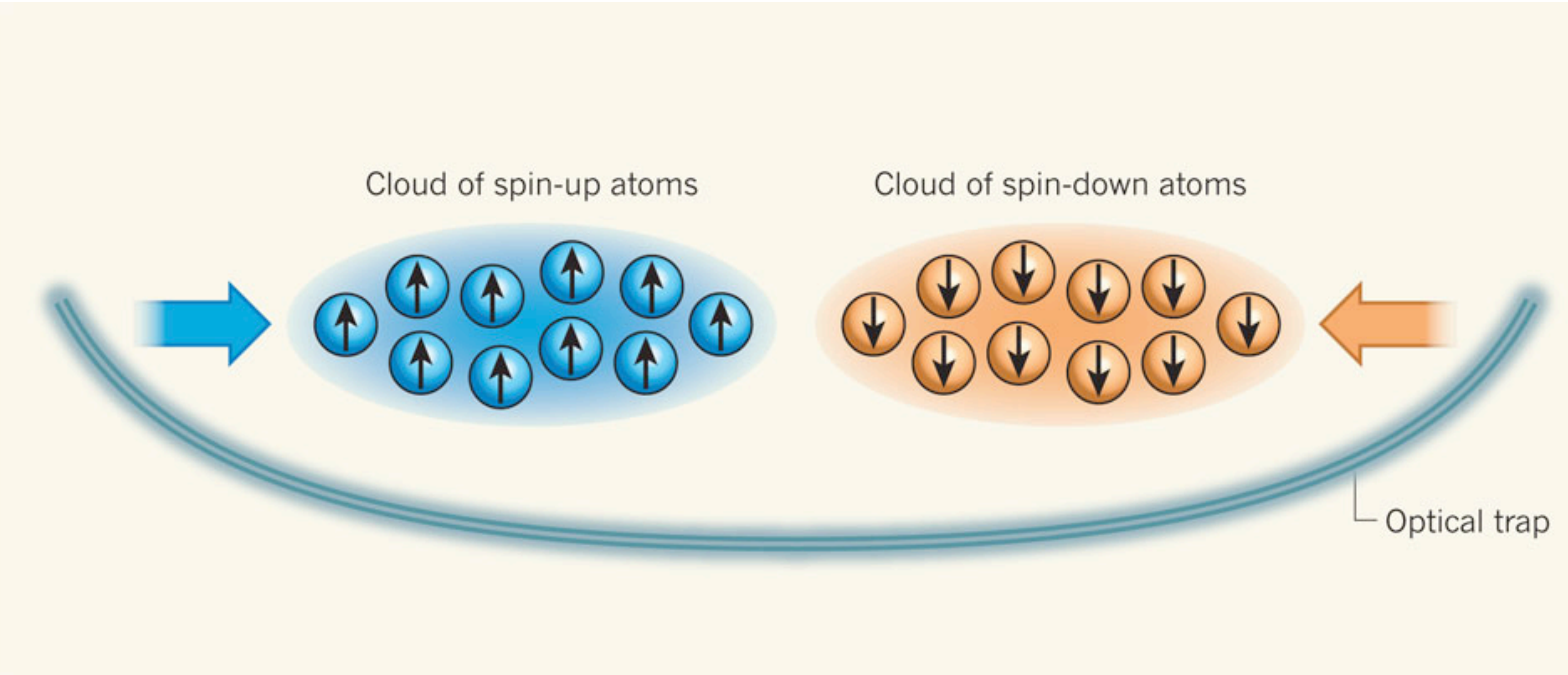
Defenu, Trombettoni, Nandori & Enss PRB 2017

Low temperature: chemical potential

- chemical potential vs interaction strength:



Boettcher, Bayha, Kedar, Murthy, Neidig, Ries, Wenz, Zürn, Jochim & Enss PRL 2016



Transport

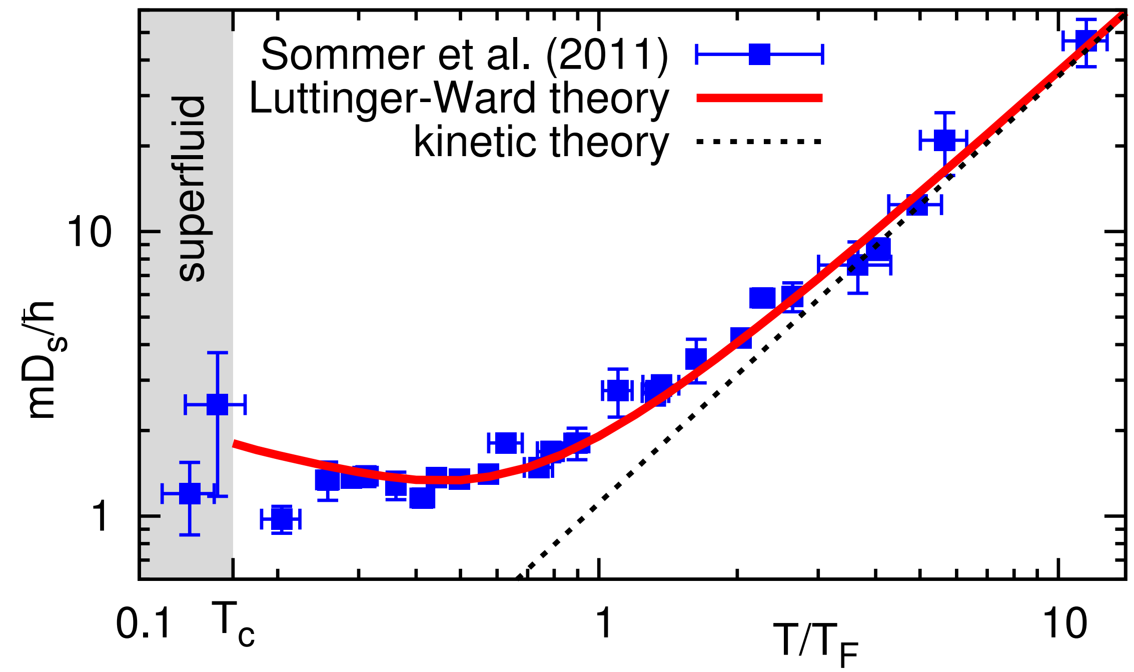
spin diffusion

Quantum bounds on transport

- 3D spin diffusion $D_s \simeq \tau_r v^2/3$:

quantum limited $$D_s \gtrsim \frac{\hbar}{m}$$

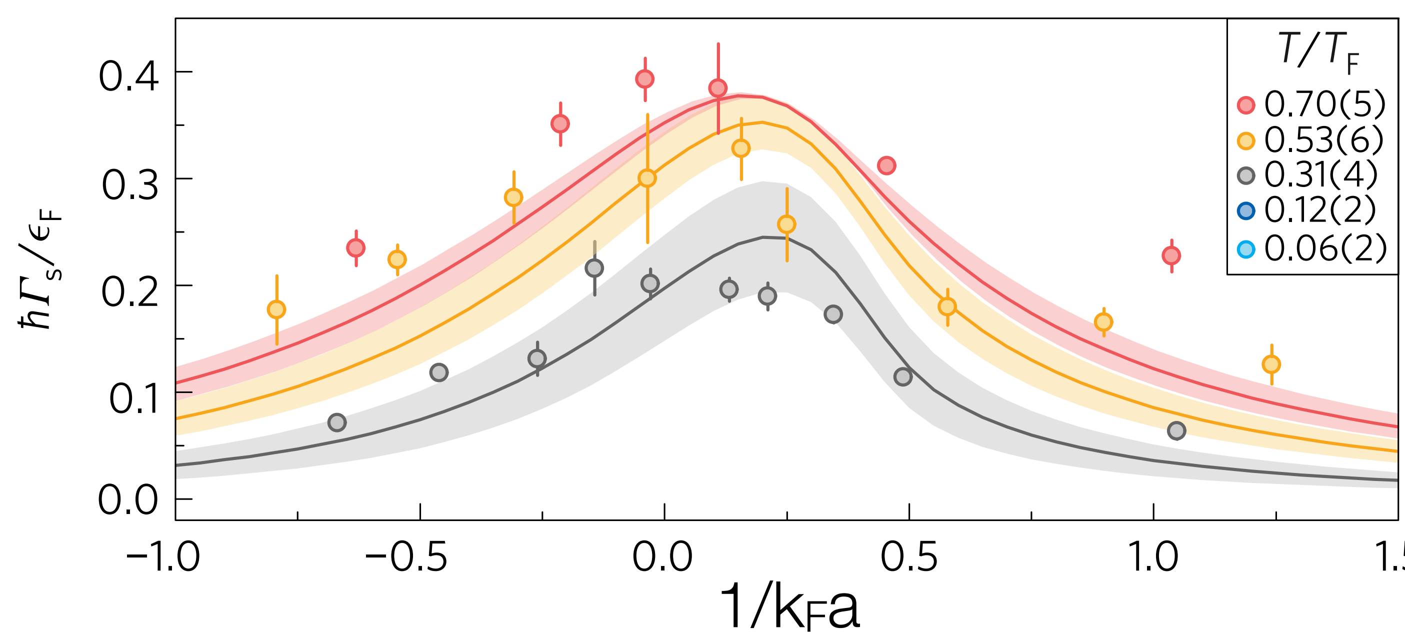
Enss & Haussmann PRL 2012



- 3D spin drag rate: $\Gamma_s = \frac{n}{m\chi_s D_s}$ **a**

quantum critical pt. $$\Gamma_s \lesssim \frac{k_B T}{\hbar}$$

Sachdev 1999



Valtolina, Scazza, Amico, Burchianti, Recati,
Enss, Inguscio, Zaccanti & Roati, Nature Phys. 2017

Quantum bounds in 2D

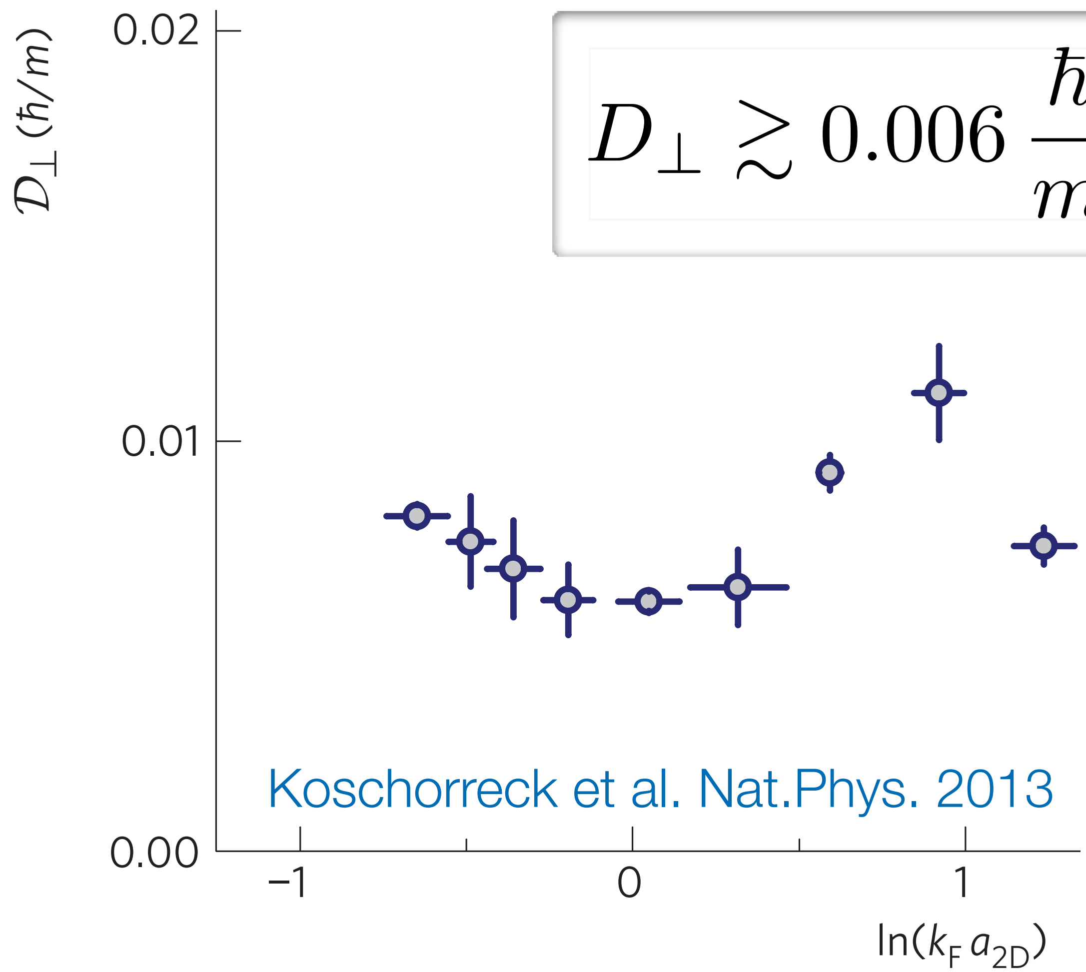
- 3D unitary Fermi gas **strongly interacting, scale invariant, quantum critical point (QCP): transport bounds**

- 2D Fermi gas **strong contact correlations,**
  but **not scale invariant, no interacting QCP:**
  transport bounds?

  2D transport bounds found for
  charge conductivity, ...

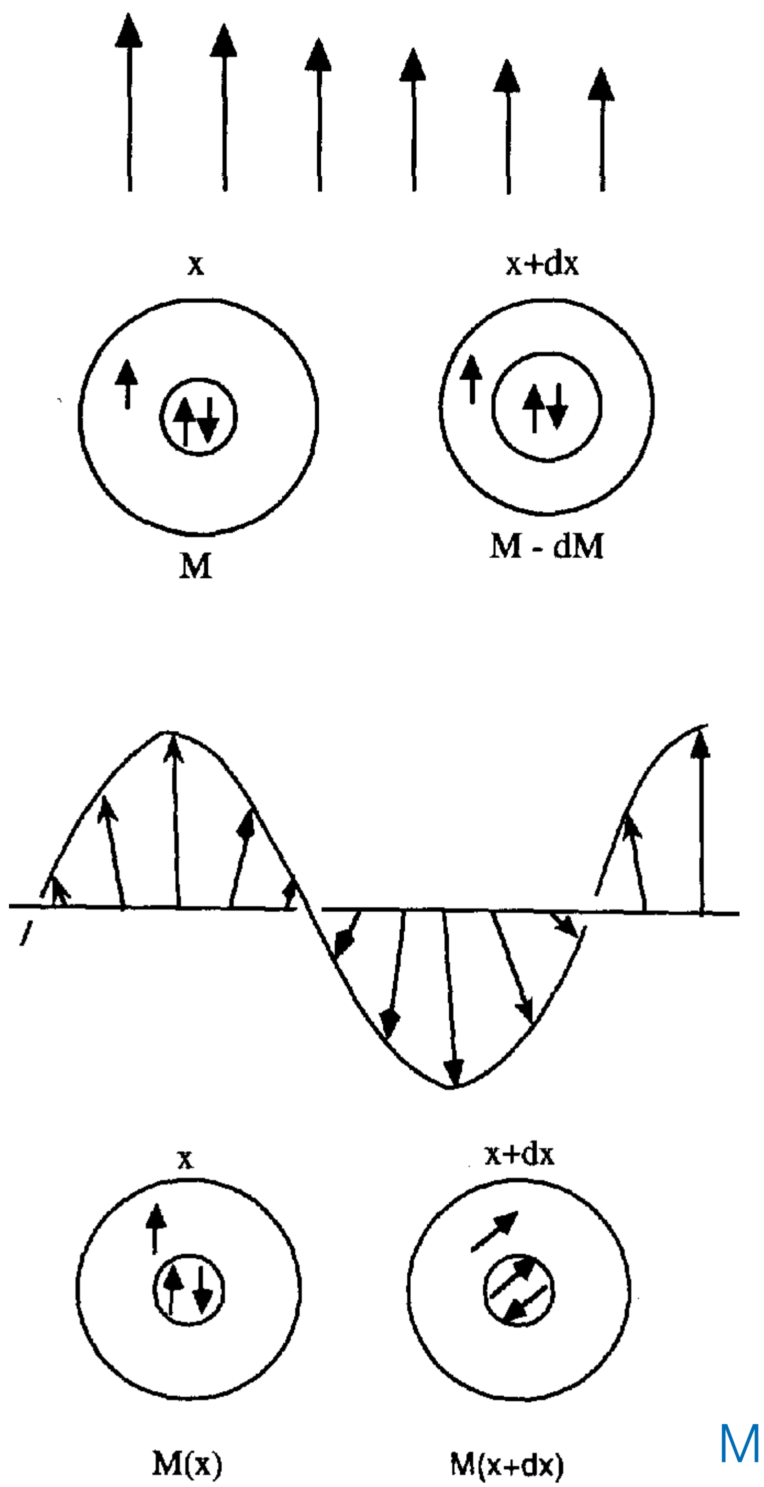
  transverse spin diffusion:

$$D_\perp \gtrsim 0.006 \frac{\hbar}{m}$$

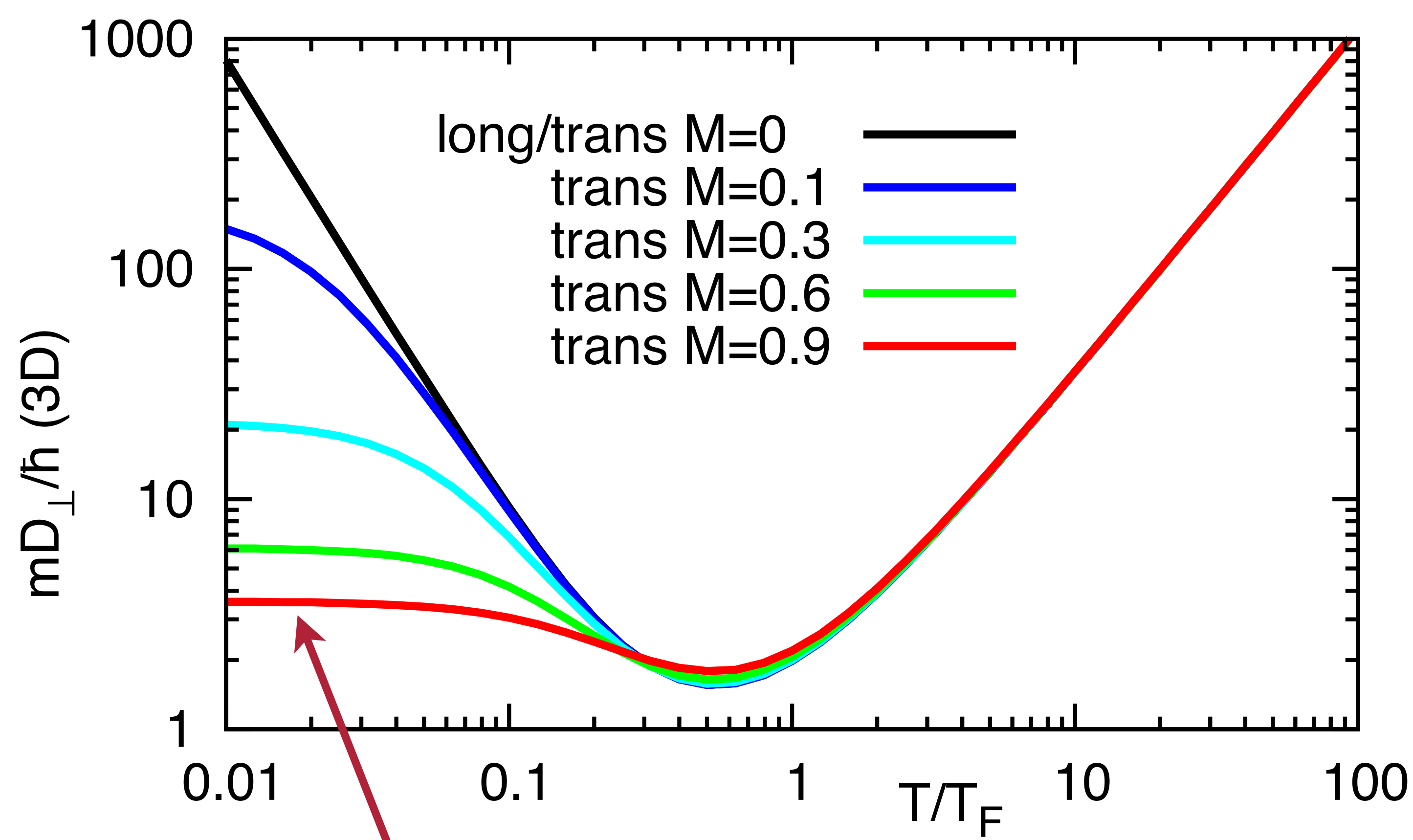


Koschorreck et al. Nat.Phys. 2013

Longitudinal vs transverse spin diffusion



Mullin & Jeon JLTP 1992



Enss PRA 2013

spin polarized &
quantum degenerate

Spin diffusion in kinetic theory

- local magnetization vector and gradient

$$\boldsymbol{\mathcal{M}}(\boldsymbol{r},t) = \mathcal{M}(\boldsymbol{r},t)\,\hat{\boldsymbol{e}}(\boldsymbol{r},t) \qquad \frac{\partial \boldsymbol{\mathcal{M}}}{\partial r_i} = \frac{\partial \mathcal{M}}{\partial r_i}\hat{\boldsymbol{e}} + \mathcal{M}\frac{\partial \hat{\boldsymbol{e}}}{\partial r_i}$$

- Boltzmann equation for spin distribution function

longitudinal

$$\frac{D\boldsymbol{\sigma}_p}{Dt} \equiv \frac{\partial \boldsymbol{\sigma}_p}{\partial t} - \sum_i v_{pi}\frac{\partial \mathcal{M}}{\partial r_i}\hat{\boldsymbol{e}}\sum_\sigma t_\sigma \frac{\partial n_{p\sigma}}{\partial \varepsilon_p}$$

$$+ \sum_i v_{pi}\frac{\partial \hat{\boldsymbol{e}}}{\partial r_i}(n_{p+} - n_{p-}) + \boldsymbol{\Omega}\times\boldsymbol{\sigma}_p = \left(\frac{\partial \boldsymbol{\sigma}_p}{\partial t}\right)_{\text{coll}}$$

transverse **spin rotation**

Landau 1956, Silin 1957;
Leggett & Rice 1968-70;
Lhuillier & Laloë 1982;
Meyerovich 1985;
Jeon & Mullin 1988, 1992

- **many-body T-matrix** in collision integral and spin rotation Enss PRA 2013
derived as leading order in large-N expansion Enss PRA 2012

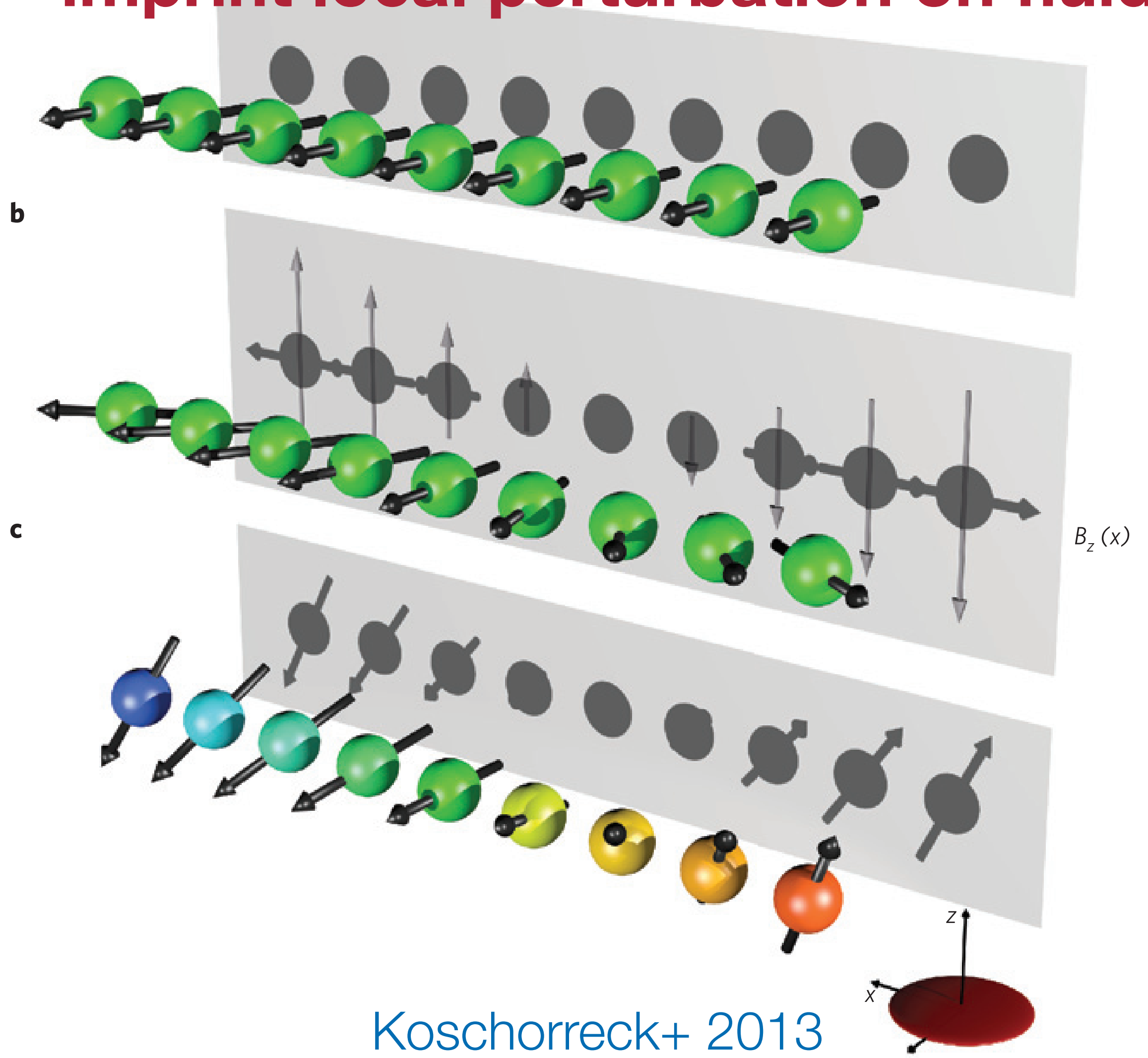
Demagnetization dynamics by spin transport

- **transverse spin current** precesses around local magnetization

$$\boldsymbol{J}_j^\perp = -D_{\mathrm{eff}}^\perp \nabla_j \boldsymbol{M} - \gamma \boldsymbol{M} \times D_{\mathrm{eff}}^\perp \nabla_j \boldsymbol{M}$$

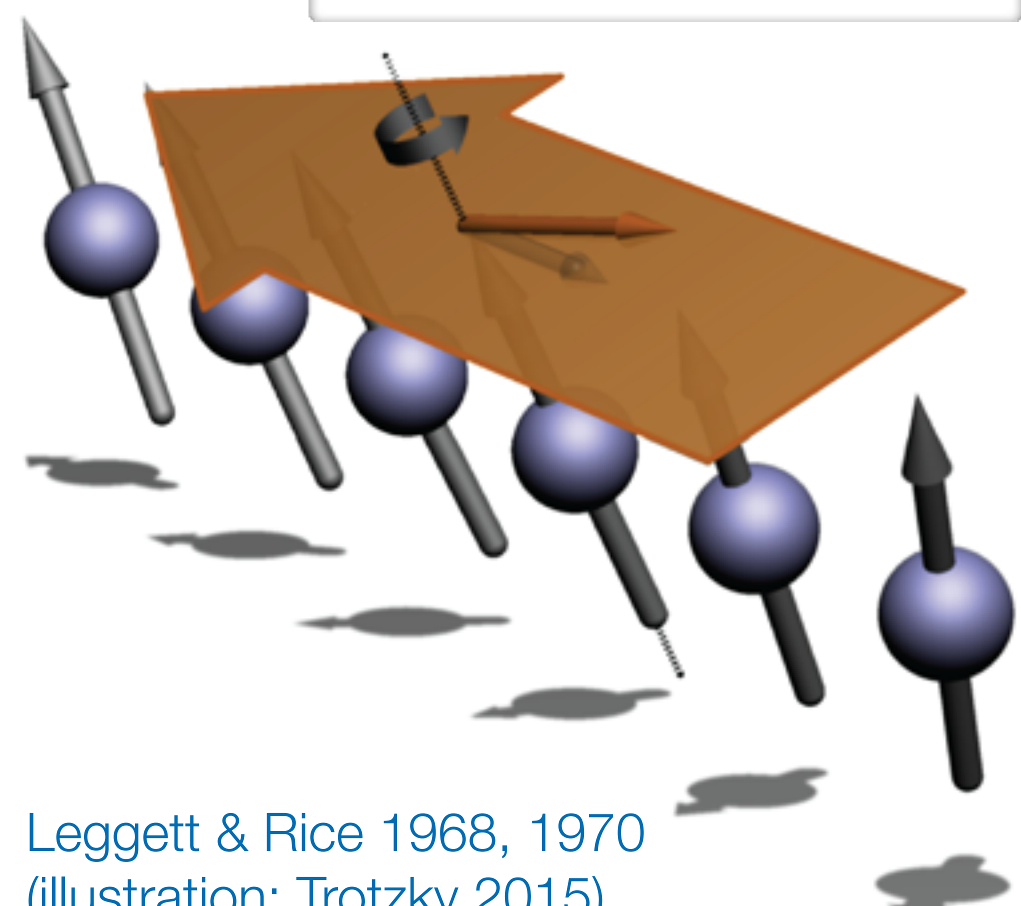
diffusive **reactive (Leggett-Rice)**

- **imprint local perturbation on fluid:**



Koschorreck+ 2013

$$D_{\mathrm{eff}}^\perp = \frac{D_0^\perp}{1 + \gamma^2 M^2}$$


Leggett & Rice 1968, 1970
(illustration: Trotzky 2015)

Demagnetization dynamics: Leggett-Rice

$$M_{xy} \equiv M_x + iM_y = i\sin(\theta)$$

$$\partial_t M_{xy} = -i\alpha x_1 M_{xy} + D_{\text{eff}}^{\perp}(1 + i\gamma M_z)\nabla_1^2 M_{xy}$$

gradient **complex diffusion**

homogeneous system: rotating frame $M_{xy}(\boldsymbol{x}, t) = e^{-i\alpha x_1 t} m(t)$

Leggett & Rice 1968, 1970

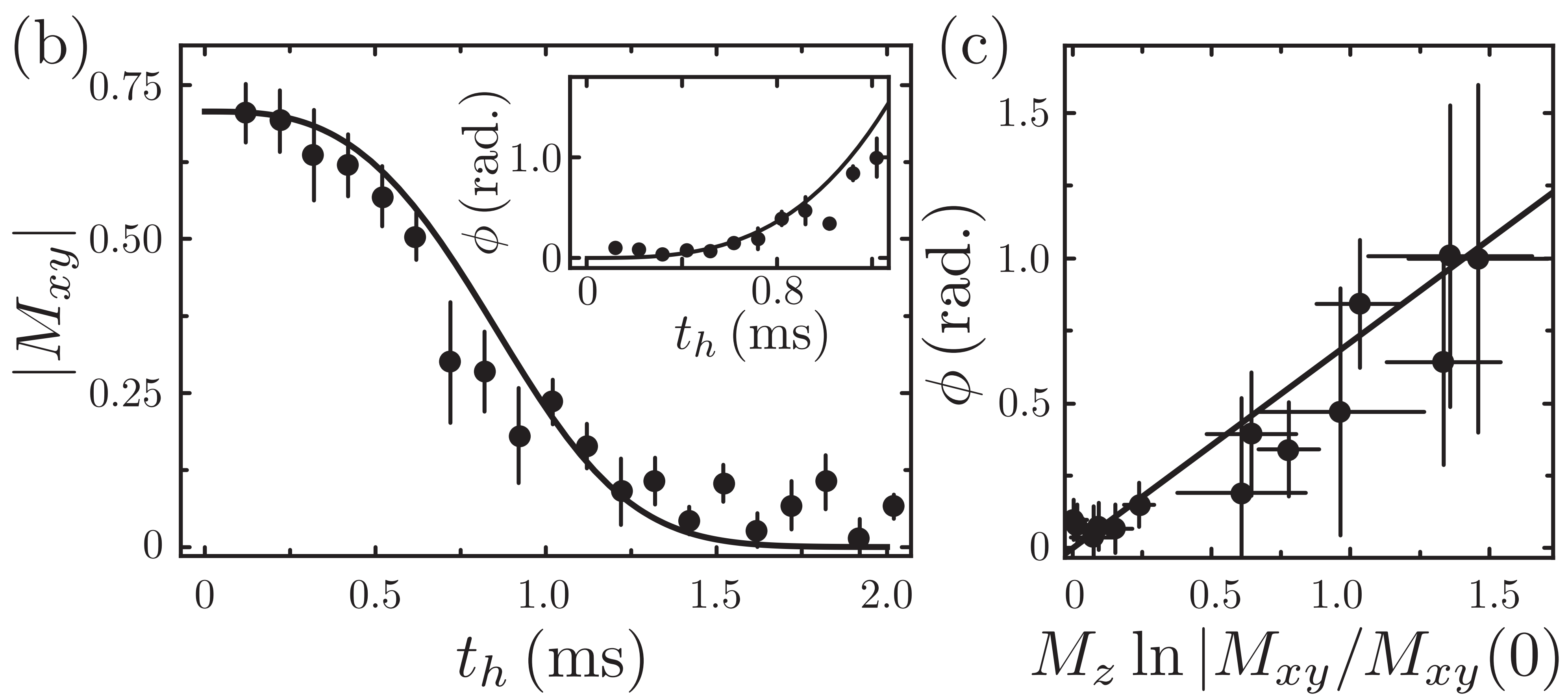
$$\partial_t m = -D_{\text{eff}}^{\perp}(1 + i\gamma M_z)\alpha^2 t^2 m(t)$$

$$M_{xy}(t) = M_{xy}(0)\, e^{-i\alpha x_1 t} e^{-D_{\text{eff}}^{\perp}(1+i\gamma M_z)\alpha^2 t^3/3}$$

$$\left|\frac{M_{xy}(t)}{M_{xy}(0)}\right| = e^{-D_{\text{eff}}^{\perp}\alpha^2 t^3/3}$$

$$\Delta\phi = \arg M_{xy} = -\gamma M_z\, D_{\text{eff}}^{\perp}\alpha^2 \frac{t^3}{3}$$

Demagnetization dynamics (Thywissen experiment)



Diffusion D_{eff} from magnitude

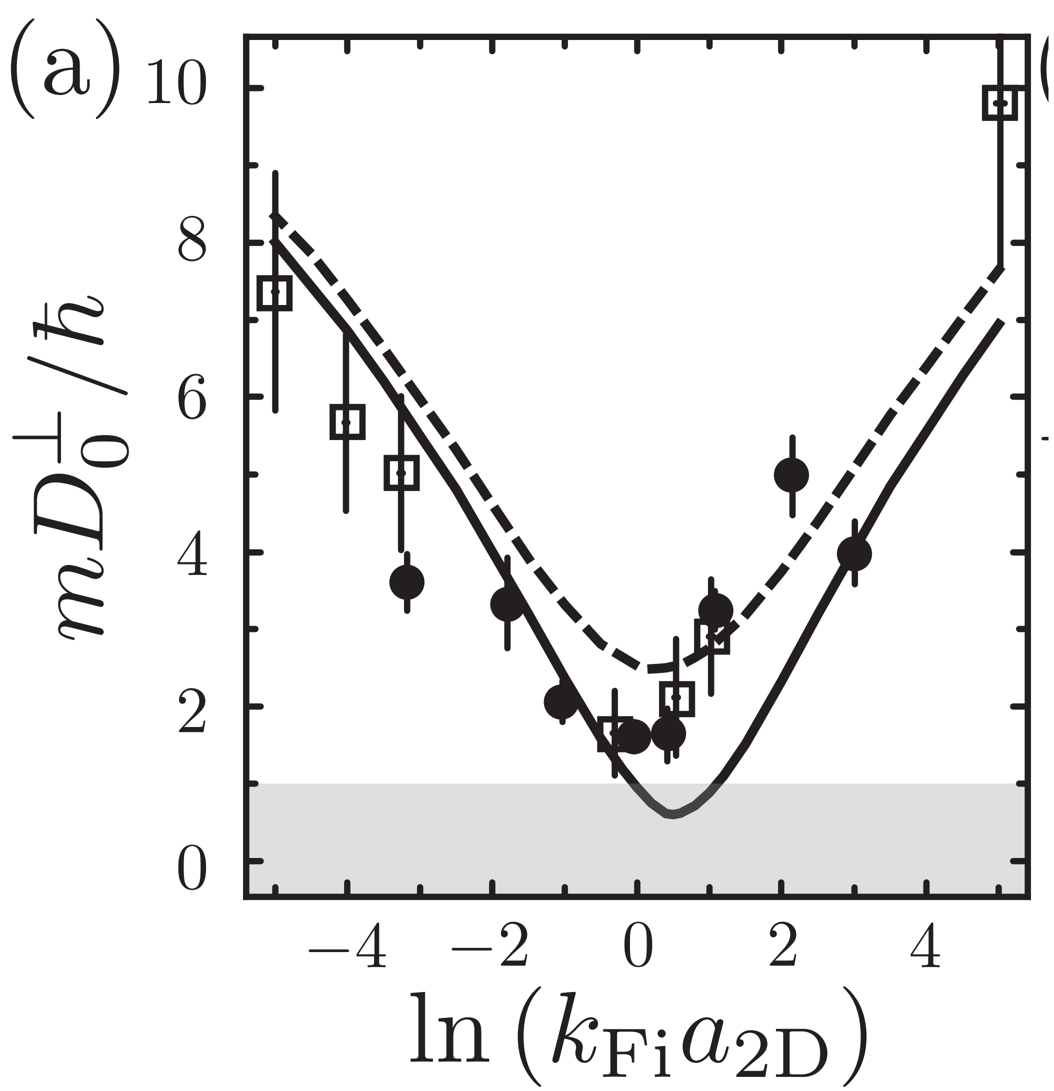
Leggett-Rice ɣ from phase

$$D_{\mathrm{eff}}^{\perp} = \frac{D_0^{\perp}}{1 + \gamma^2 M^2}$$

Luciuk, Smale, Böttcher, Sharum, Olsen, Trotzky, Enss & Thywissen, PRL 118, 130405 (2017)

Transverse diffusion

interaction dependence: minimum near unitarity,
confirm quantum limited spin diffusion



Luciuk, Smale, Böttcher, Sharum, Olsen, Trotzky, Enss & Thywissen, PRL 118, 130405 (2017)

$D_0^\perp = 1.7(6)\ \hbar/m$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4k}\left|\frac{2\pi}{i\frac{\pi}{2} - \ln(ka_{2D})}\right|^2 \leq \frac{4}{k}$$

transport calculation:

1. compute spin transport coefficient from microscopic quantum theory
2. solve Boltzmann equation for spin helix in trapping potential

cf. Enss PRA 2015

Spin-rotation parameter ɣ

- precession of spin current around local magnetization m:

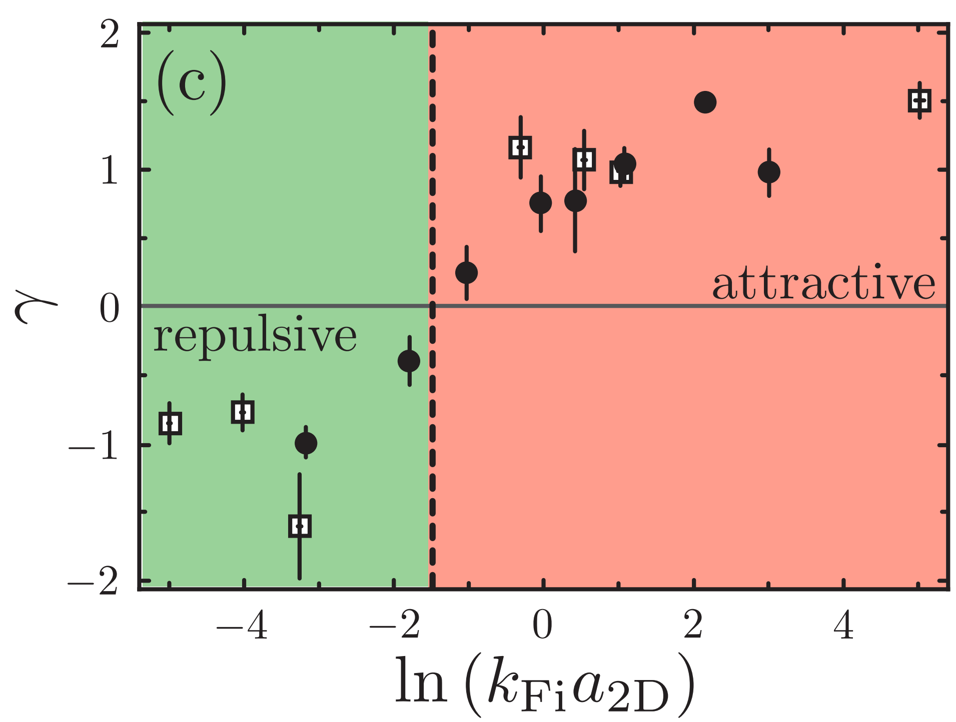
Ramsey phase ϕ ∝ $\gamma M = - W m \frac{\tau_\perp}{\hbar}$ W: effective interaction

molecular field

interaction dependence:
zero crossing near ln(k_Fa)=-1

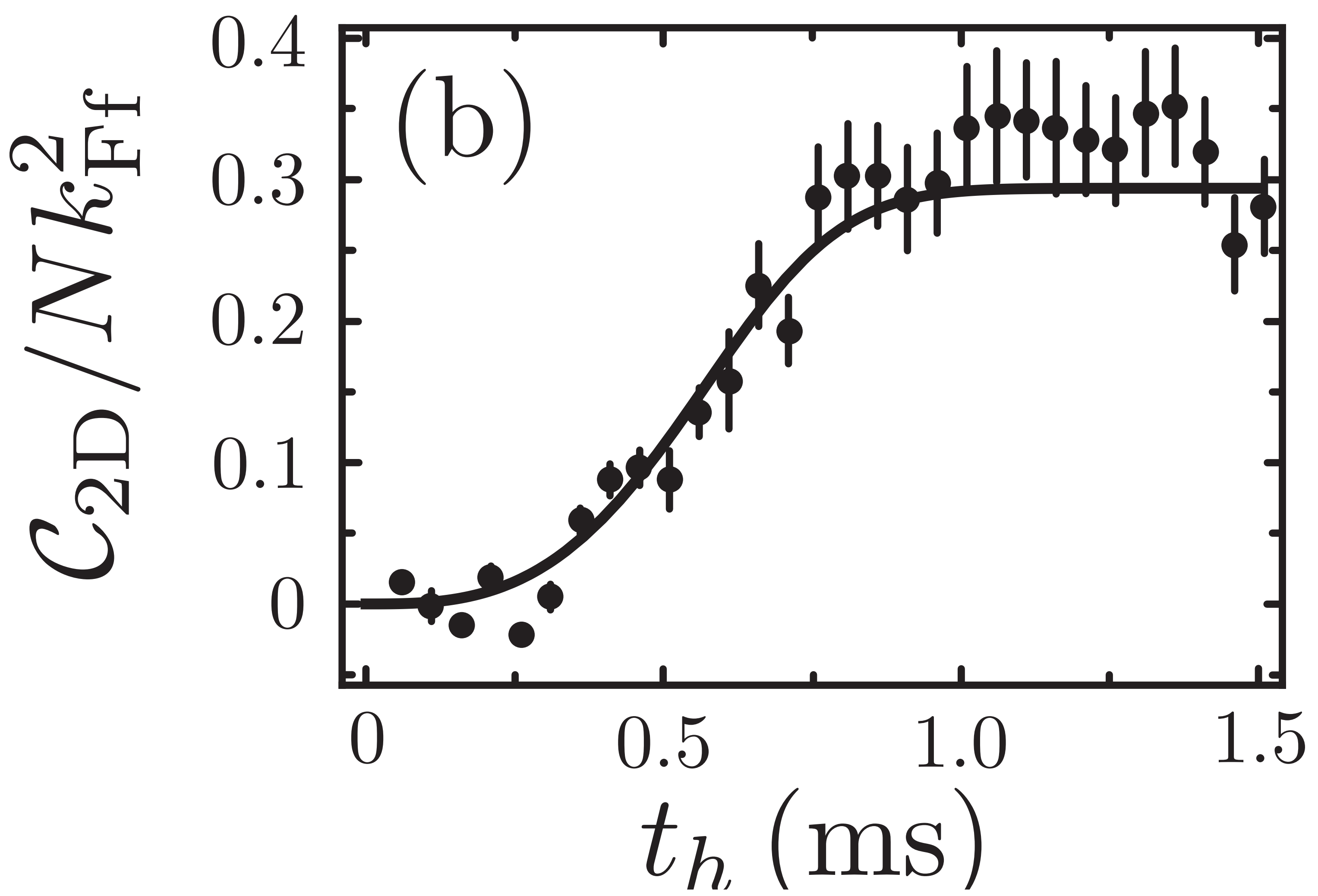
$$\mathrm{Re} f = - \frac{2\pi \ln(ka)}{\pi^2/4 + \ln^2(ka)}$$

repulsive interaction
for ln(k_Fa) < -1



attractive interaction
for ln(k_Fa) > -1

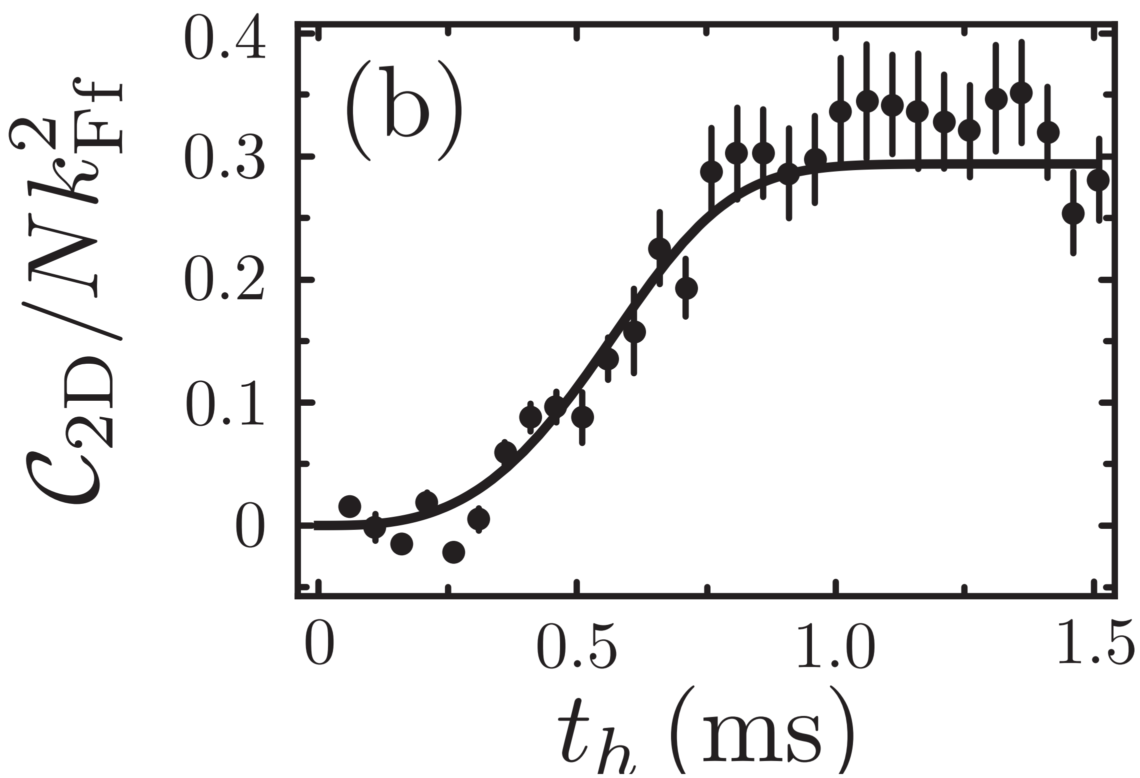
Local correlations build up over time

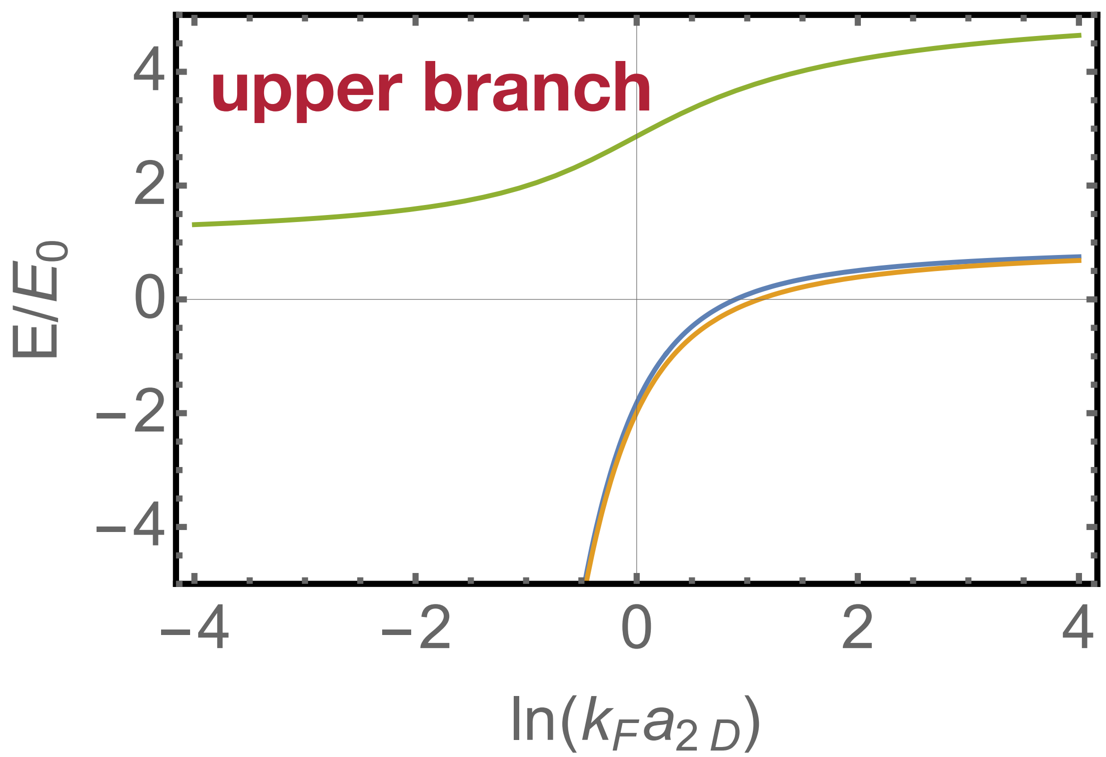


local correlations build up during demagnetization

Luciuk+ PRL 2017

Local correlations build up over time





contact 15x smaller than in ground state:

Fröhlich+ PRL 2012: C~5 N k_F^2

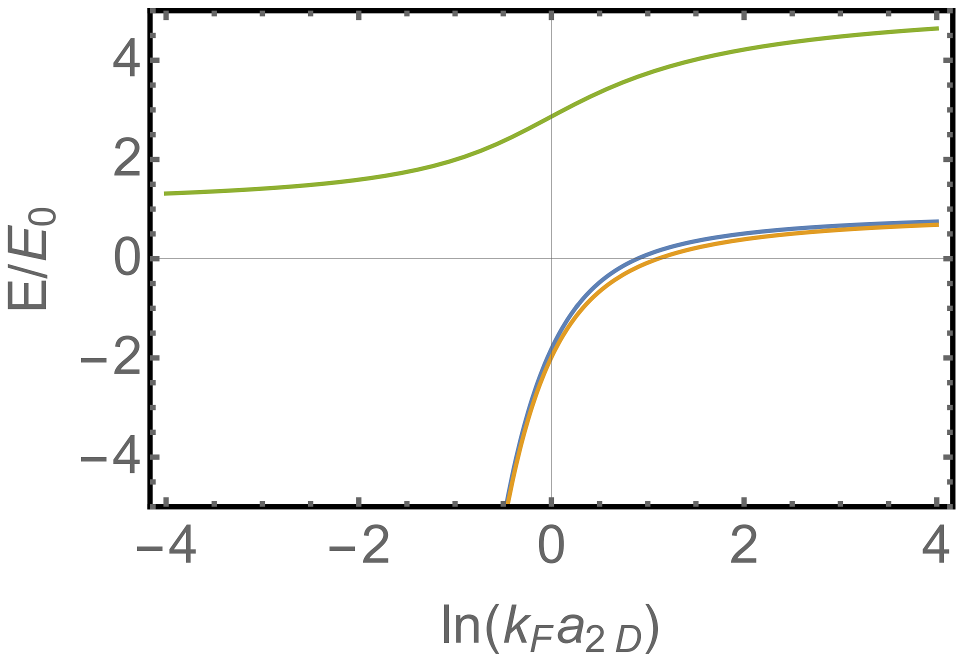
demagnetization into excited state which is almost scale invariant

Luciuk+ PRL 2017

upper branch stable ≫ Fermi time

Small reheating during demagnetization

demagnetization switches on interaction, but onto lower or upper branch?



3. **reheating: total E conserved;**
   initially T/TF=0.3 (polarized gas)
   lower branch T/TF=2.5 @ ln(kFa)=0
   (application of EoS)

   measured much smaller T/TF=0.7

initial polarized gas (scale invariant):

$$V = \frac{E}{2}$$

after demagnetization (contact measures scale invariance breaking):

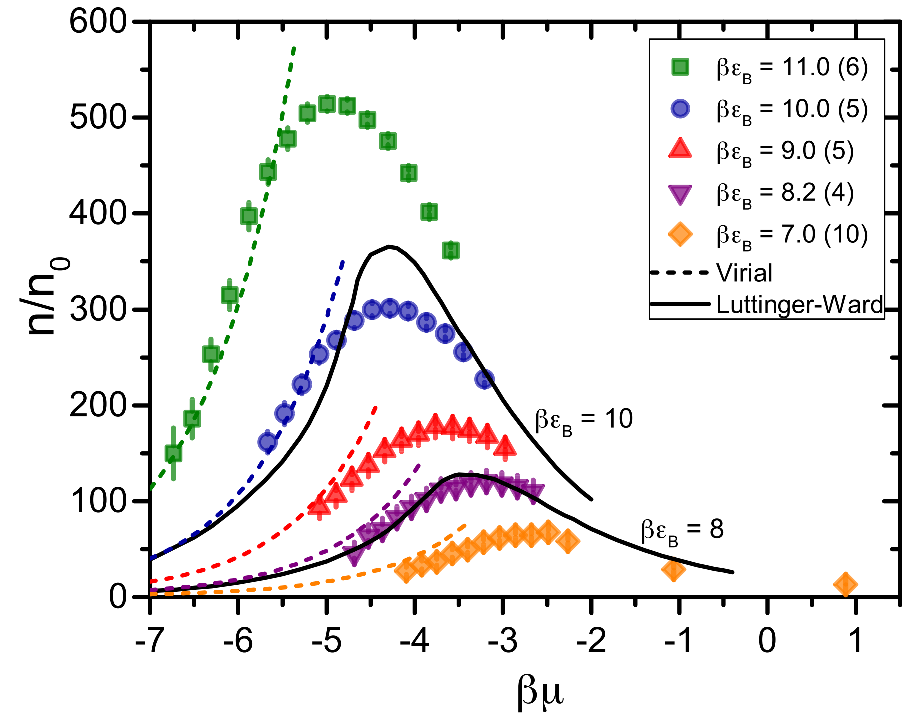
$$V = \frac{1}{2}E + \frac{\hbar^2}{8\pi m}\mathcal{C}_{2\mathrm{D}}$$

virial (cloud size) grows only 4% during demagnetization

$$V/N = \frac{1}{2}m(\omega_1^2\langle x_1^2\rangle + \omega_2^2\langle x_2^2\rangle)$$

Conclusion

- **2D equation of state at T=0 and T>0:**
  EoS strongly scale dependent
  density driven crossover from Bose to Fermi
  substantial density renormalization
  Bauer, Parish & Enss PRL 2014
  Boettcher *et al.* PRL 2016

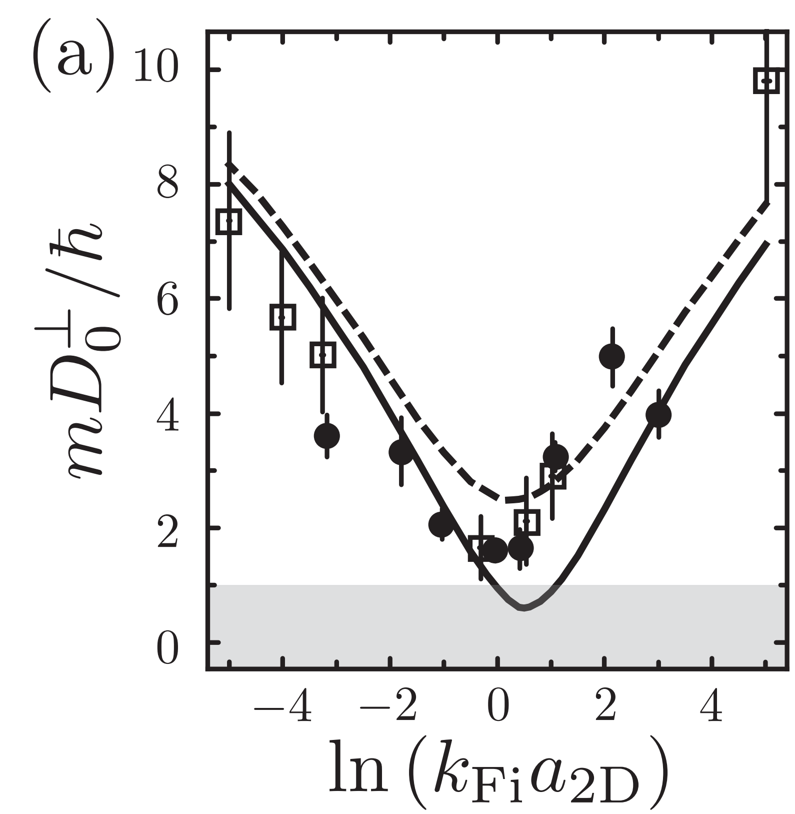


- **spin transport in strongly interacting gas:**
  quantum bound relaxation rate

$$D_0^\perp \gtrsim \frac{\hbar}{m}$$

$$\tau_r^{-1} \lesssim \frac{k_B T}{\hbar}$$

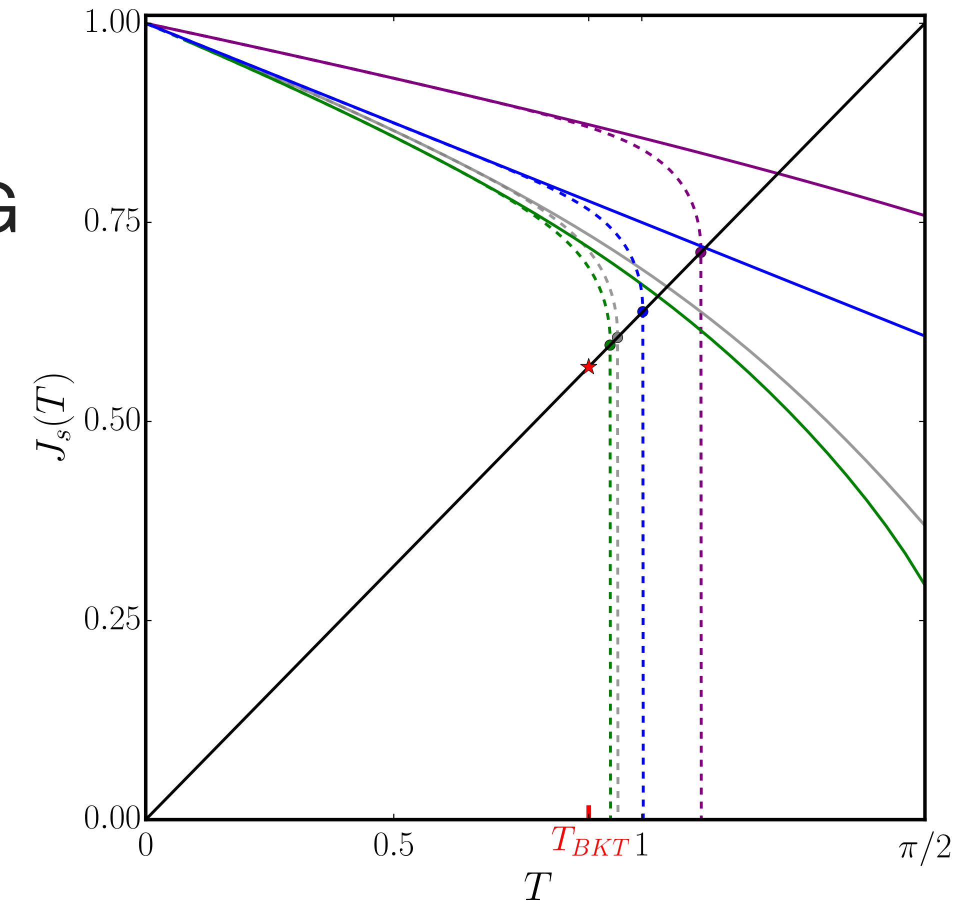
scale invariance for transport almost recovered
Luciuk, Smale, Böttcher, Sharum, Olsen, Trotzky,
Enss & Thywissen PRL 2017; Enss PRA 2015 & 2013



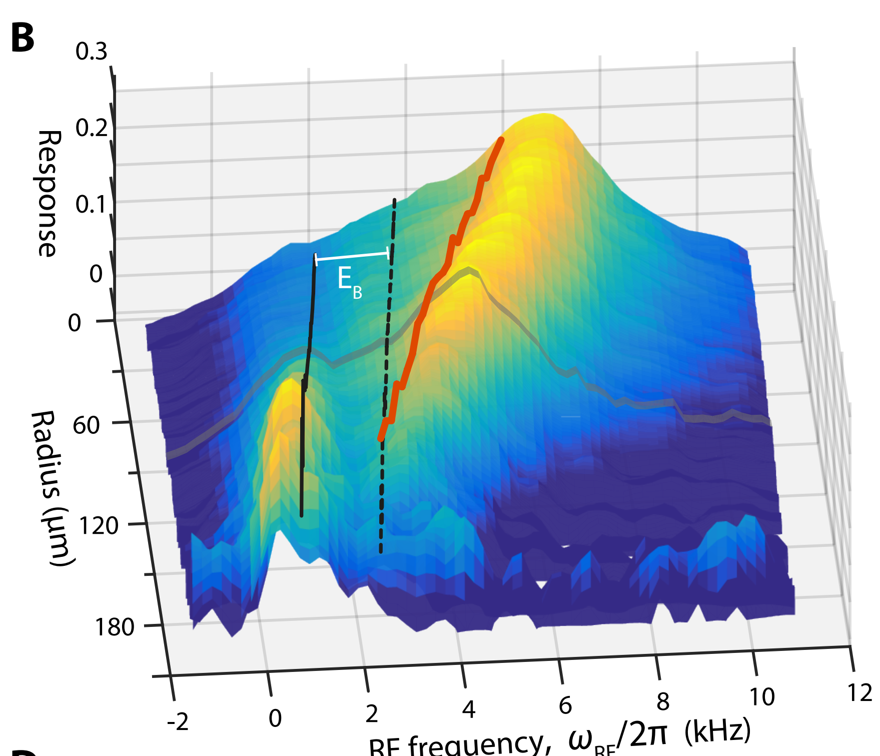
- **upper branch physics:**
  demagnetization into metastable upper branch

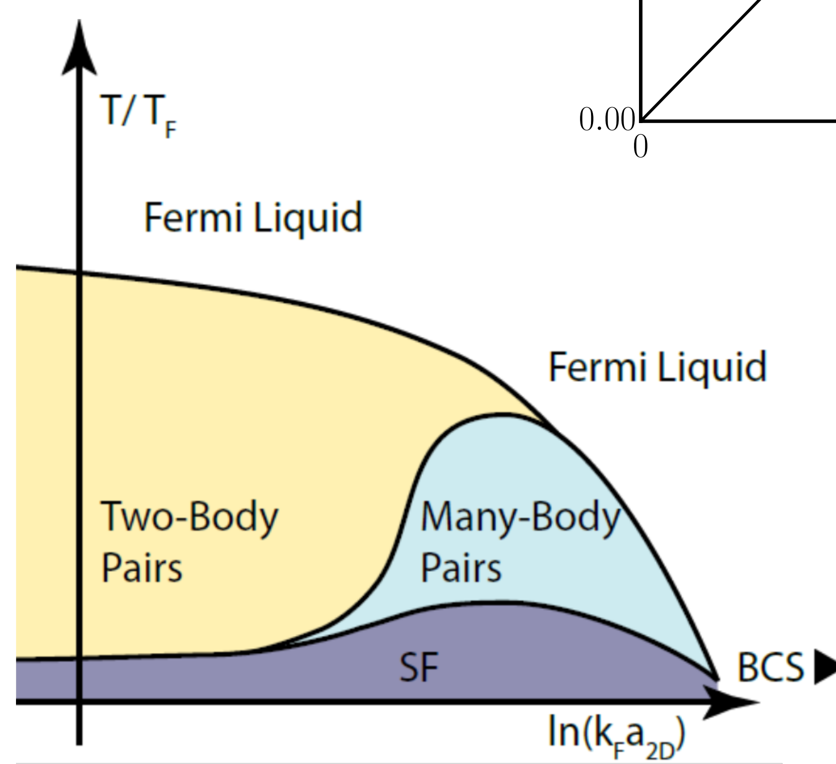
Outlook

- **Kosterlitz-Thouless transition:**
  vortex, spin wave and **density (amplitude)** fluctuations on equal footing using functional RG
  Defenu, Trombettoni, Nandori & Enss PRB **96**, 174505 (2017)



- **pairing fluctuations in normal phase:**
  from two- to many-body pairing above Tc
  Murthy, Neidig, Klemt, Bayha, Boettcher, Enss, Holten, Zürn, Preiss & Jochim, 1705.10577





Additional material

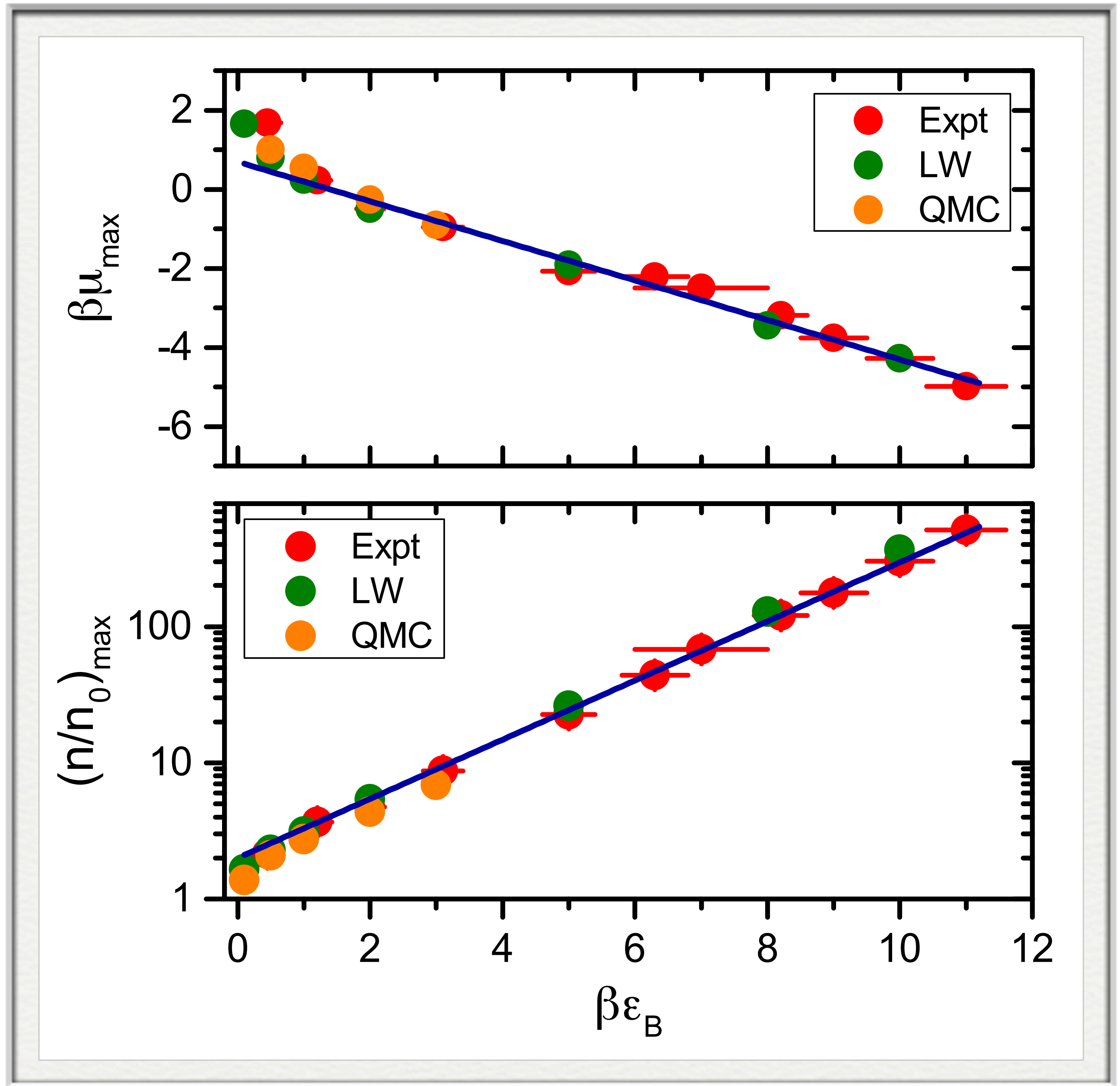
Scaling of density maximum n/n0

- **maximum** where $\tilde{\mu} \simeq 0$:

$$(\beta\mu)_{\max} \simeq -\frac{\beta\varepsilon_B}{2} + \ln 2$$

at density

$$(n/n_0)_{\max} \simeq 2e^{\beta\varepsilon_B/2}$$



Boettcher+ PRL 2016