Universal quantum transport in ultracold Fermi gases

How slowly can spins diffuse?

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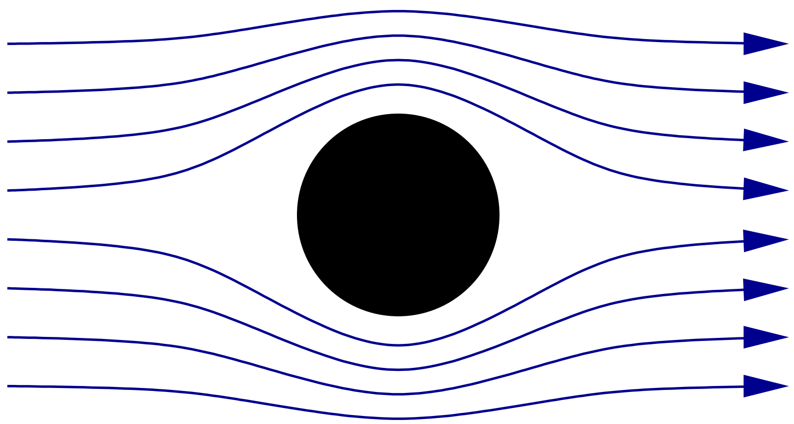
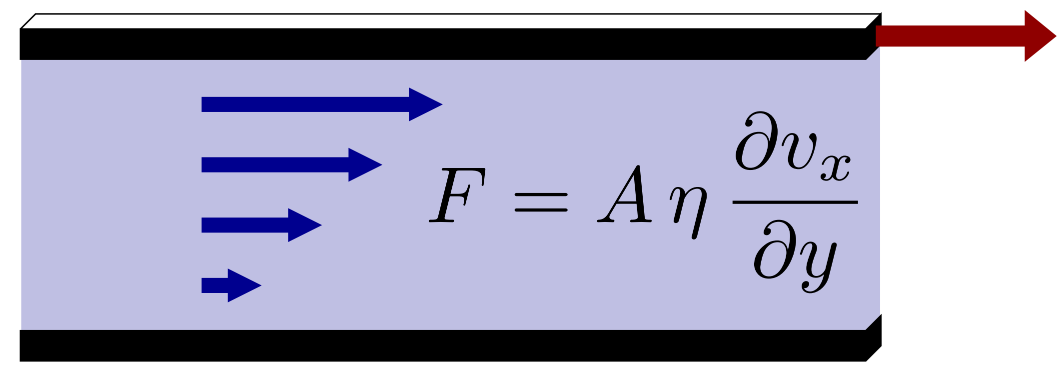
Transport in quantum fluids

Schäfer & Teaney 2009

can mass flow without friction?





shear viscosity to entropy ratio: expt minimum values $\eta/s \sim 0.4 \ldots 0.8\,\hbar/k_B$

kinetic theory: $\frac{\eta}{s} \sim \frac{\ell_{\mathrm{mfp}}}{\ell}\,\frac{\hbar}{k_B} \gtrsim \mathcal{O}(1)\,\frac{\hbar}{k_B}$ extrapolated from weak coupling

gauge-gravity duality: $\boldsymbol{\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}}$ **perfect fluidity** Kovtun, Son & Starinets 2005
conjectured as universal lower bound

transport near quantum critical point (QCP): incoherent relaxation

Strongly interacting Fermi gas

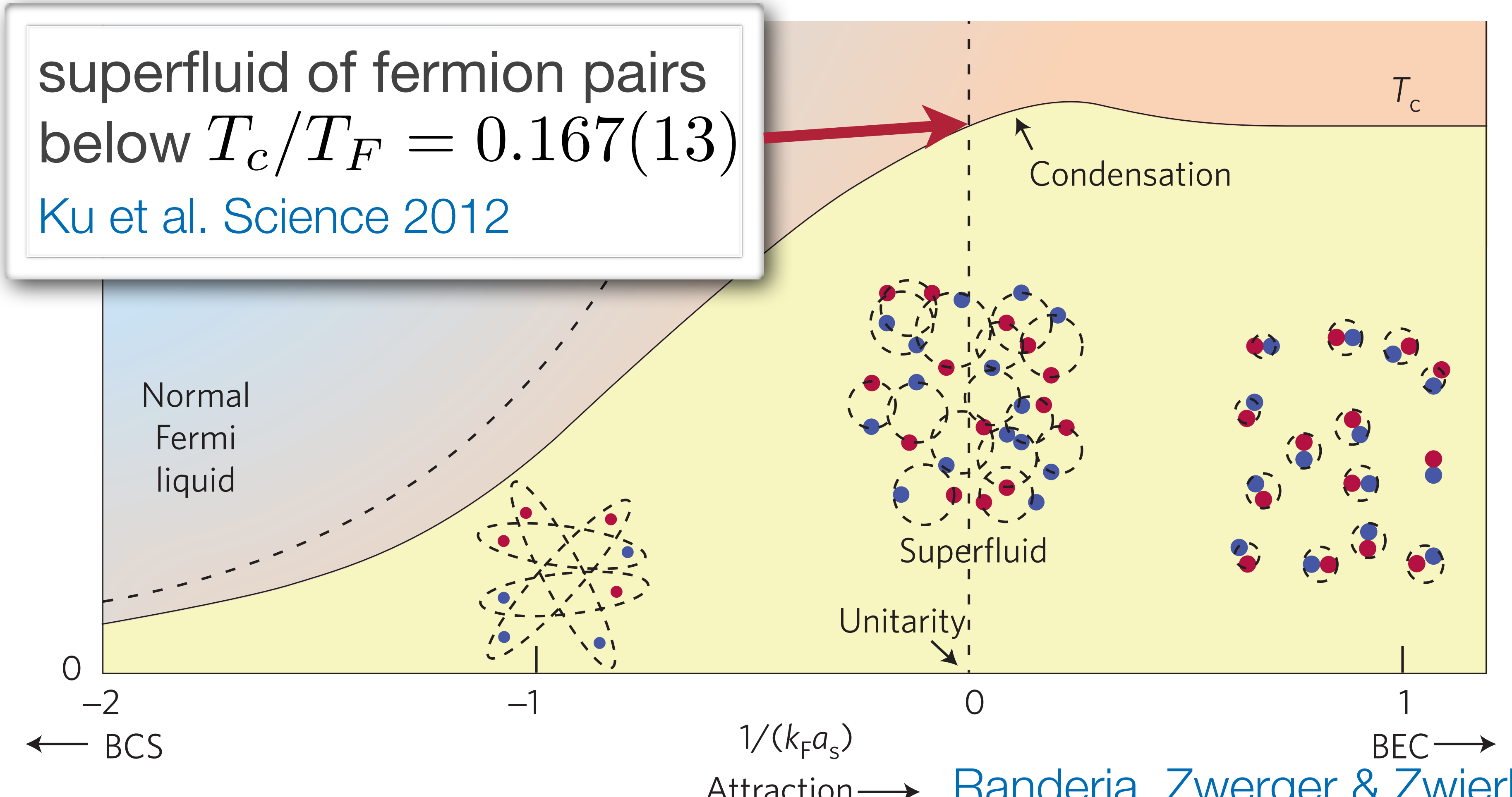
Giorgini, Pitaevskii & Stringari 2008
Bloch, Dalibard & Zwerger 2008

- **dilute** gas of ↑ and ↓ fermions, $|r_0| \ll \ell$ contact interaction: **universal**

$$\mathcal{H} = \int d\mathbf{x} \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big(-\frac{\hbar^2\nabla^2}{2m} - \mu_\sigma\Big)\psi_\sigma + g_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

- **strong** s-wave scattering, $|a| \gg \ell$ (Feshbach resonance); scale invariance

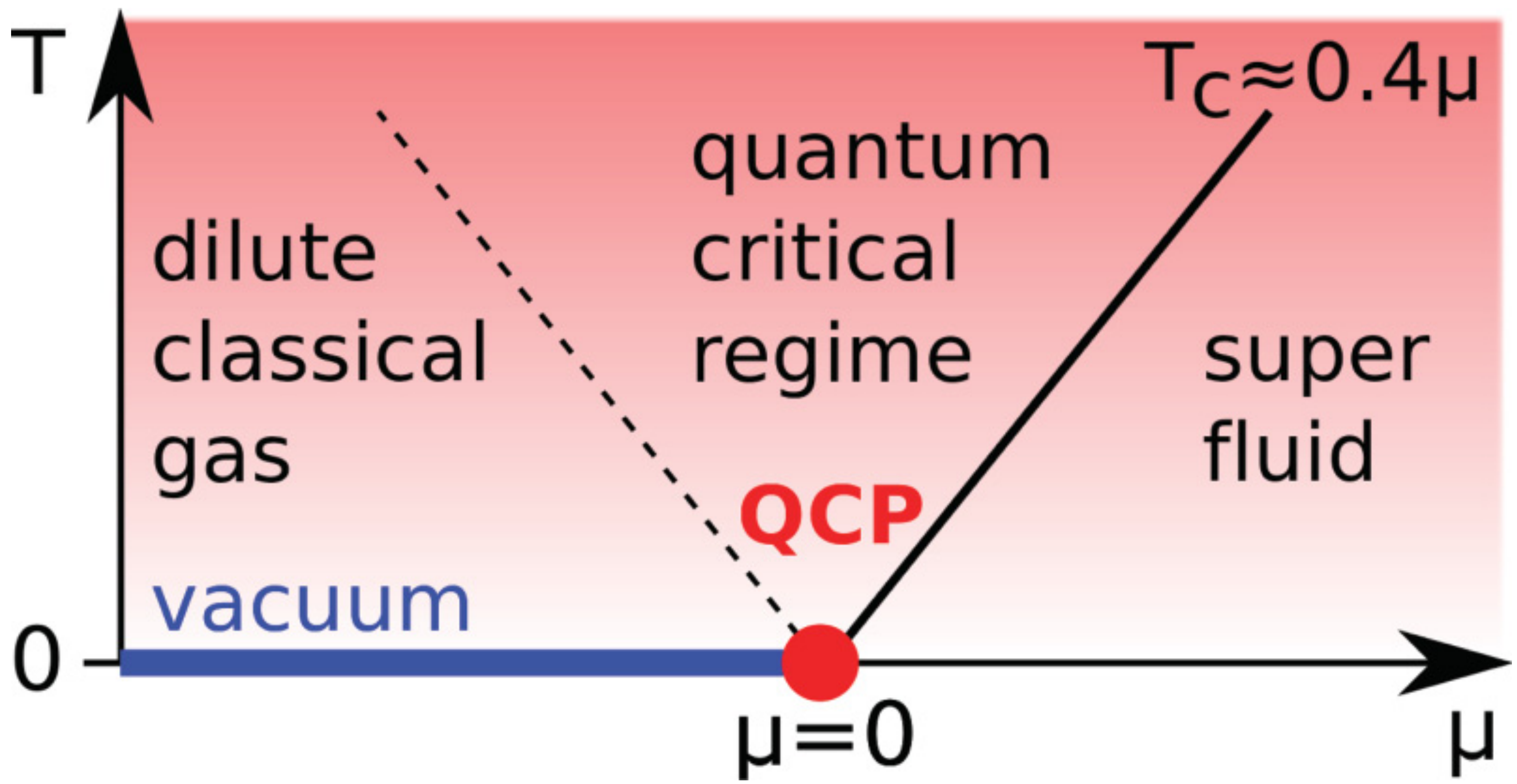
superfluid of fermion pairs below $T_c/T_F = 0.167(13)$
Ku et al. Science 2012



Randeria, Zwerger & Zwierlein 2012

Quantum critical point

- resonant fixed point is **Quantum Critical Point** (QCP) at T=0, μ=0, 1/a=0
  Nikolic & Sachdev 2007



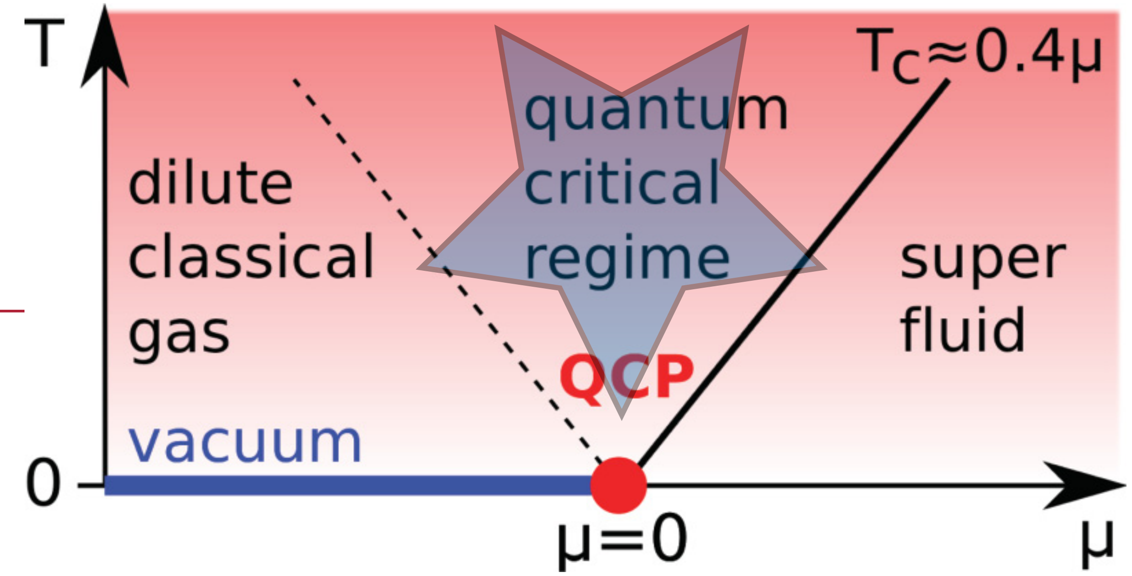
at unitarity 1/a=0

Enss PRA 2012

- abrupt change of ground state at QCP
  density n is **order parameter:** vacuum for T=0, μ<0
  gapless excitations above QCP: affect measurements in quantum critical regime

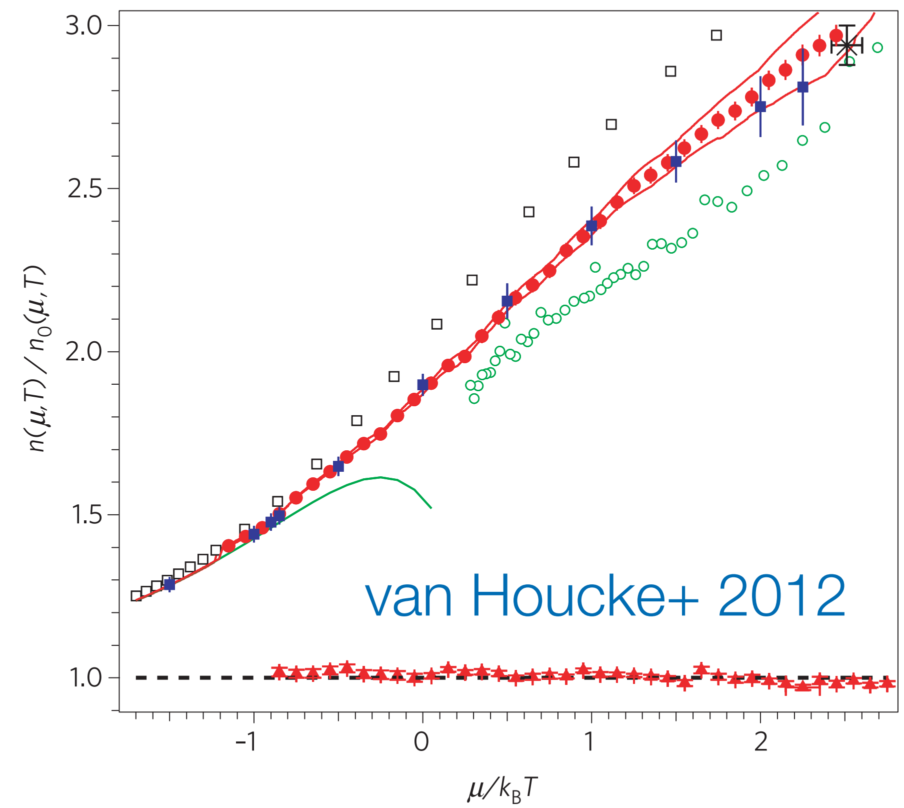
Universal properties

- at unitarity 1/a=0 **scale invariance:** properties depend only on **μ/T** (“angle”)
  Zhang+ Science 2012



- e.g. equation of state $n = \lambda_T^{-3}\, f_n(\mu/T)$

  measured by Zwierlein group (2012),
  computed using Bold Diagrammatic MC



- **quantum critical regime** above QCP:

$$\lambda_T \approx n^{-1/3} \quad (T_c \lesssim T \lesssim T_F)$$

  quantum and thermal fluctuations equally important, interplay challenging

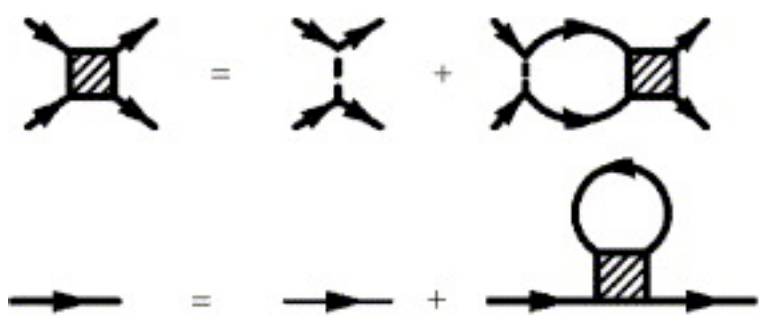
  temperature only available scale for **incoherent relaxation:** Sachdev 1999

$$\frac{\hbar}{\tau_\eta} = \mathcal{O}(1)\, k_B T \quad \Rightarrow \quad \frac{\eta}{s} = \frac{2}{5} T \tau_\eta \approx 0.7\, \frac{\hbar}{k_B} \quad (\mu = 0)$$

large-N exp’n
Enss 2012

Luttinger-Ward approach

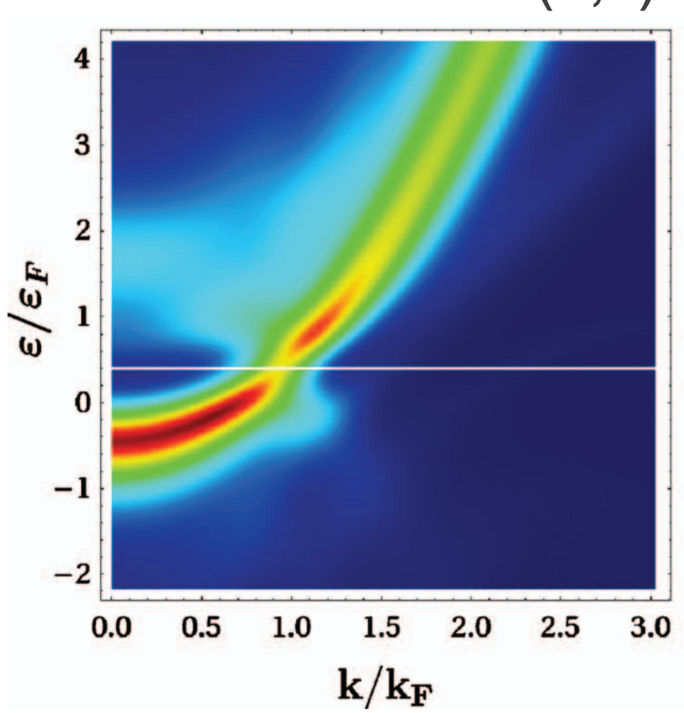
- repeated particle-particle scattering dominant in dilute gas:



self-consistent T-matrix Haussmann 1993, 1994; Haussmann et al. 2007

self-consistent fermion propagator
(300 momenta / 300 Matsubara frequencies)

- spectral function A(k,ε) at Tc



Haussmann et al. 2009

works above and below Tc;
directly in continuum limit

Tc=0.16(1) and ξ=0.36(1)
agree with experiment

conserving: exactly fulfills scale invariance and Tan relations
Enss PRA 2012

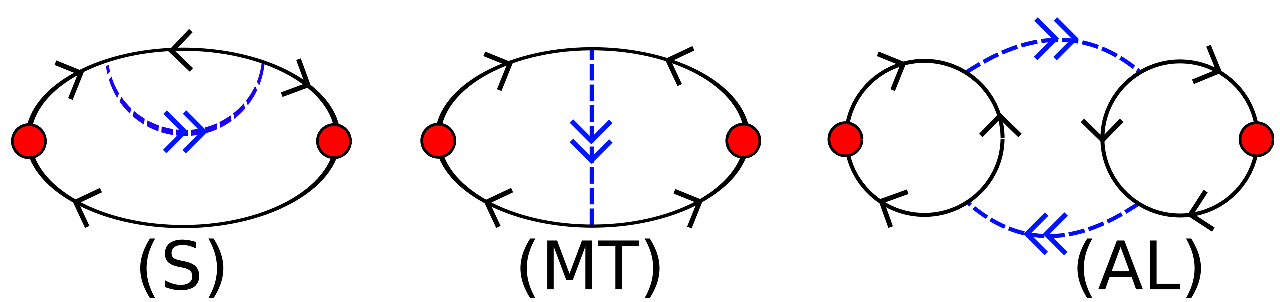
Transport in linear response

- shear viscosity from stress correlations (cf. hydrodynamics),

$$\eta(\omega) = \frac{1}{\omega} \operatorname{Re} \int_0^\infty dt\, e^{i\omega t} \int d^3x \left\langle \left[\hat{\Pi}_{xy}(\boldsymbol{x},t), \hat{\Pi}_{xy}(0,0) \right] \right\rangle$$

with stress tensor $\hat{\Pi}_{xy} = \sum_{\mathbf{p},\sigma} \frac{p_x p_y}{m} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma}$ (cf. Newton $\frac{\partial v_x}{\partial y}$)

- correlation function (Kubo formula): Enss, Haussmann & Zwerger, Annals Physics 2011

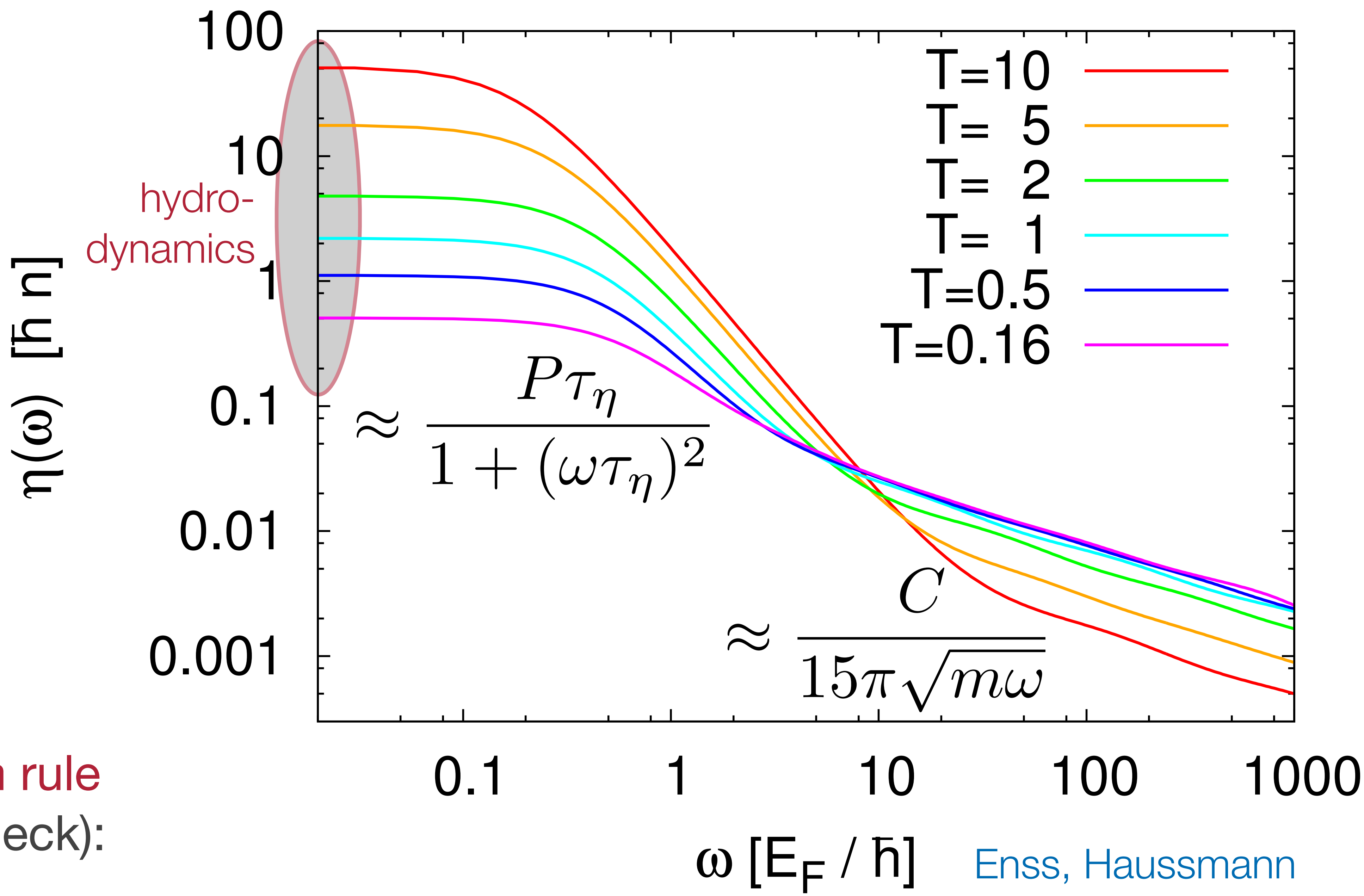
$\eta(\omega) =$ 

(resummed to infinite order)

- transport via fermions and bosonic molecules: very efficient description, satisfies conservation laws, scale invariance and Tan relations Enss PRA 2012

- assumes no quasiparticles: beyond Boltzmann kinetic theory, works near Tc; includes pseudogap **and** vertex corrections

High-energy tails in stress correlation (shear viscosity)

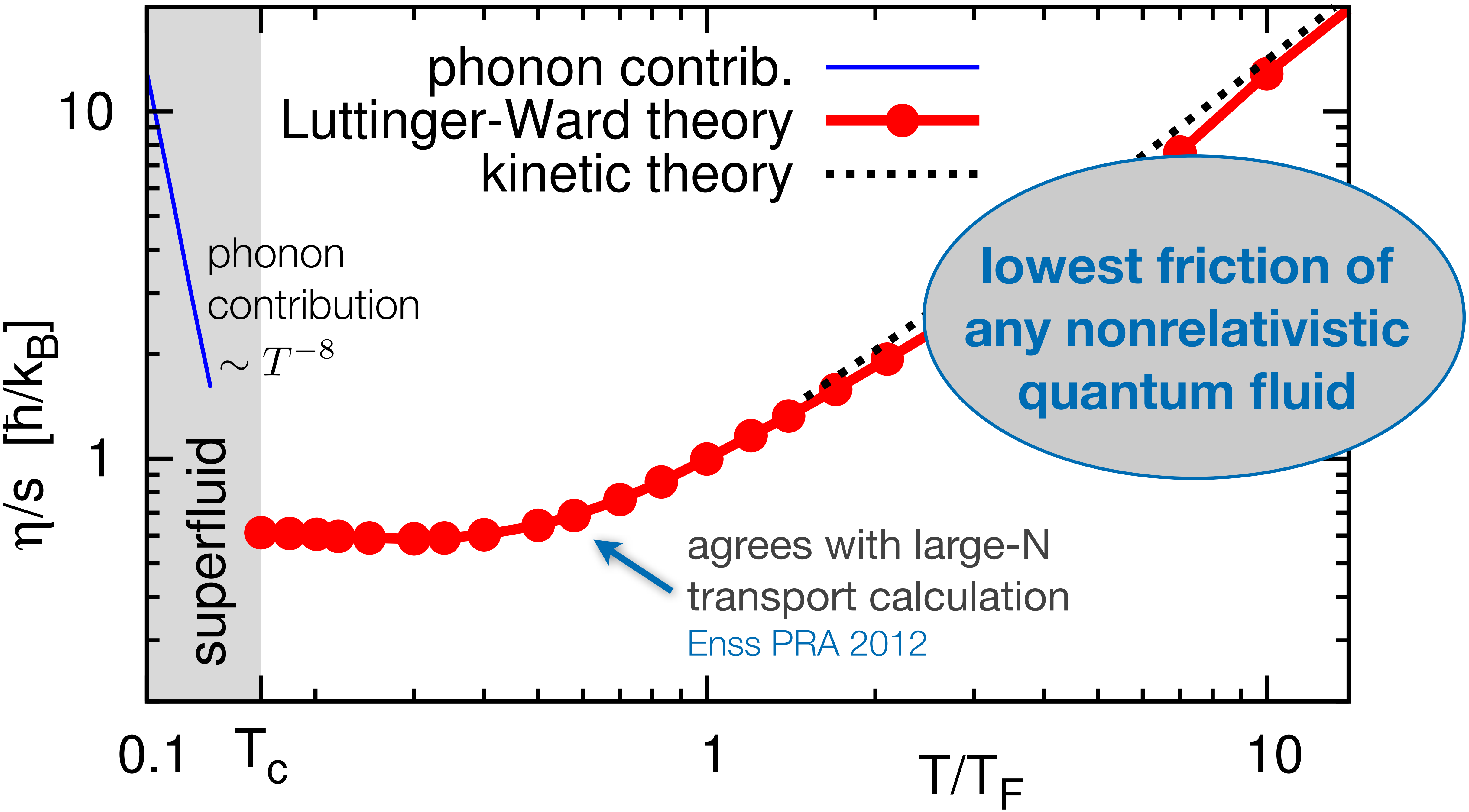


Enss, Haussmann & Zwerger 2011

exact viscosity sum rule
(nonperturbative check):

$$\frac{2}{\pi}\int_0^\infty d\omega\ [\eta(\omega) - \text{tail}] = P - \frac{C}{4\pi m a}$$

Taylor & Randeria 2010; Enss, Haussmann & Zwerger 2011; Enss 2013



cf. theory: Bruun, Massignan, Schäfer, Smith, ...

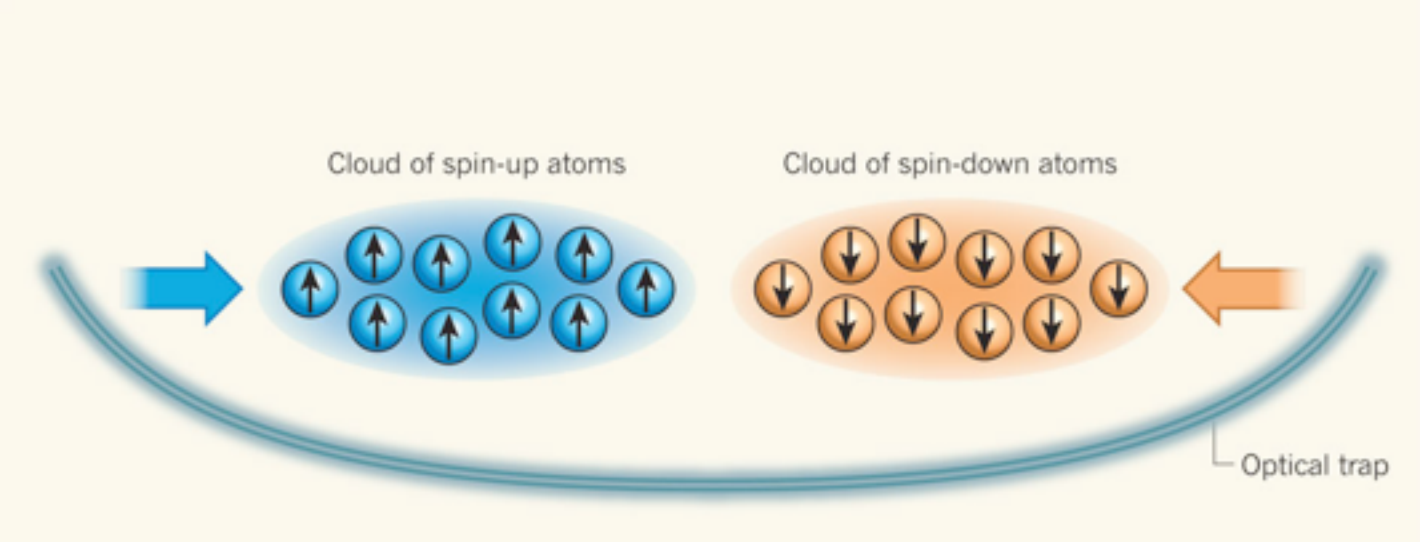
experiment: Cao+ Science 2011

Shear viscosity/entropy of the unitary Fermi gas

Enss, Haussmann & Zwerger 2011

Spin transport with ultracold gases

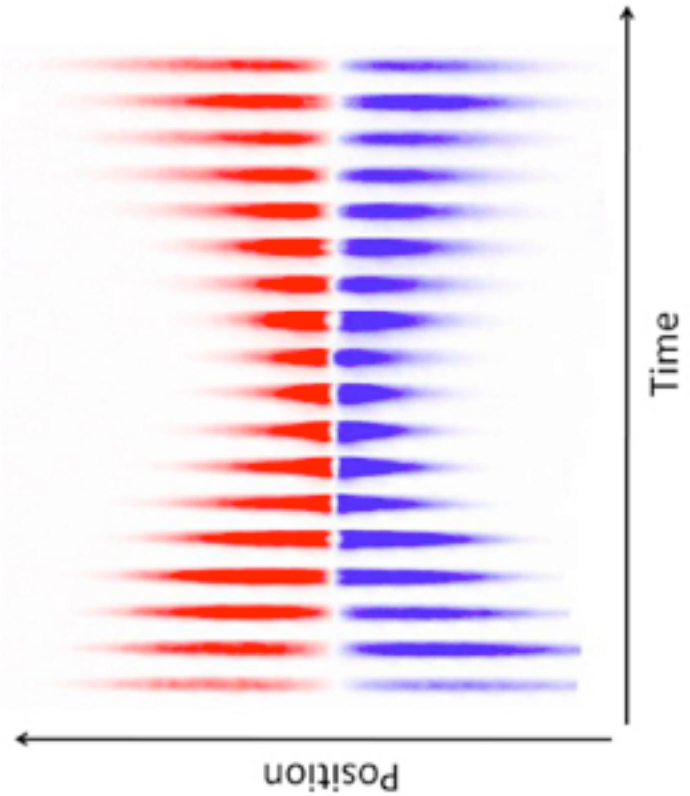
- experiment: spin-polarized clouds in harmonic trap



picture: J. Thomas 2011

- strongly interacting gas [movie courtesy Martin Zwierlein]:

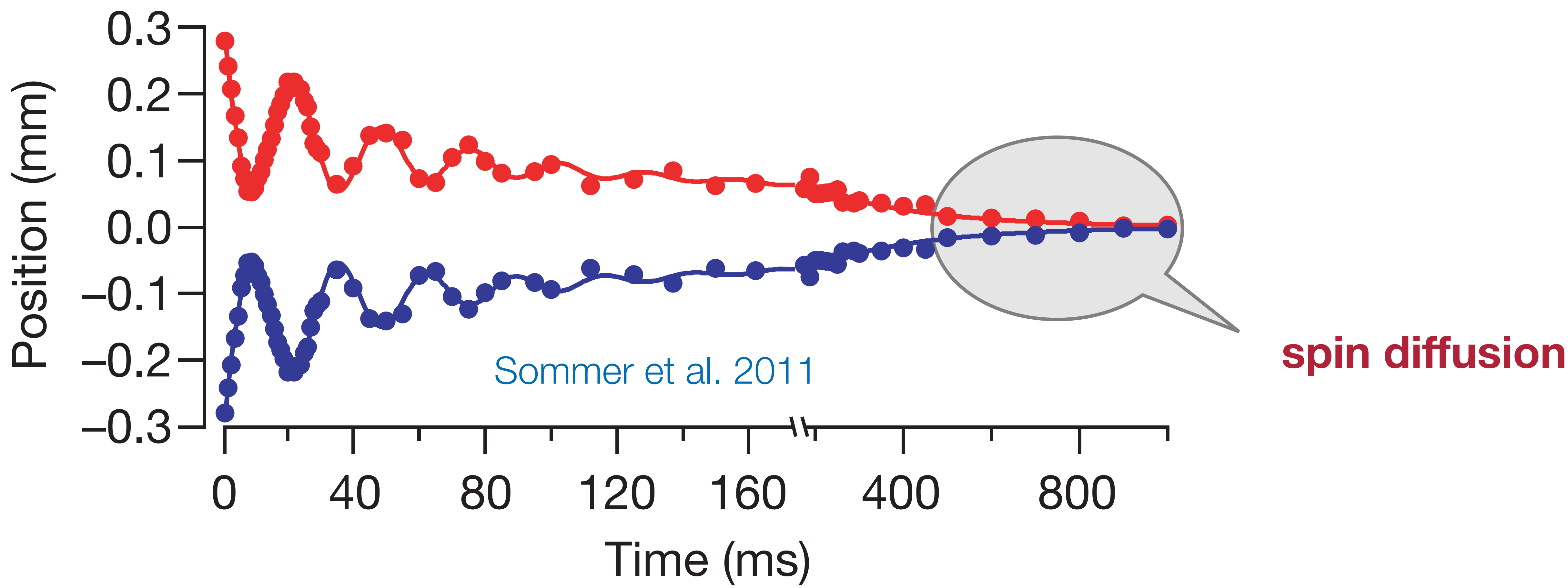
bounce!



A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature **472**, 201 (2011)

Spin diffusion

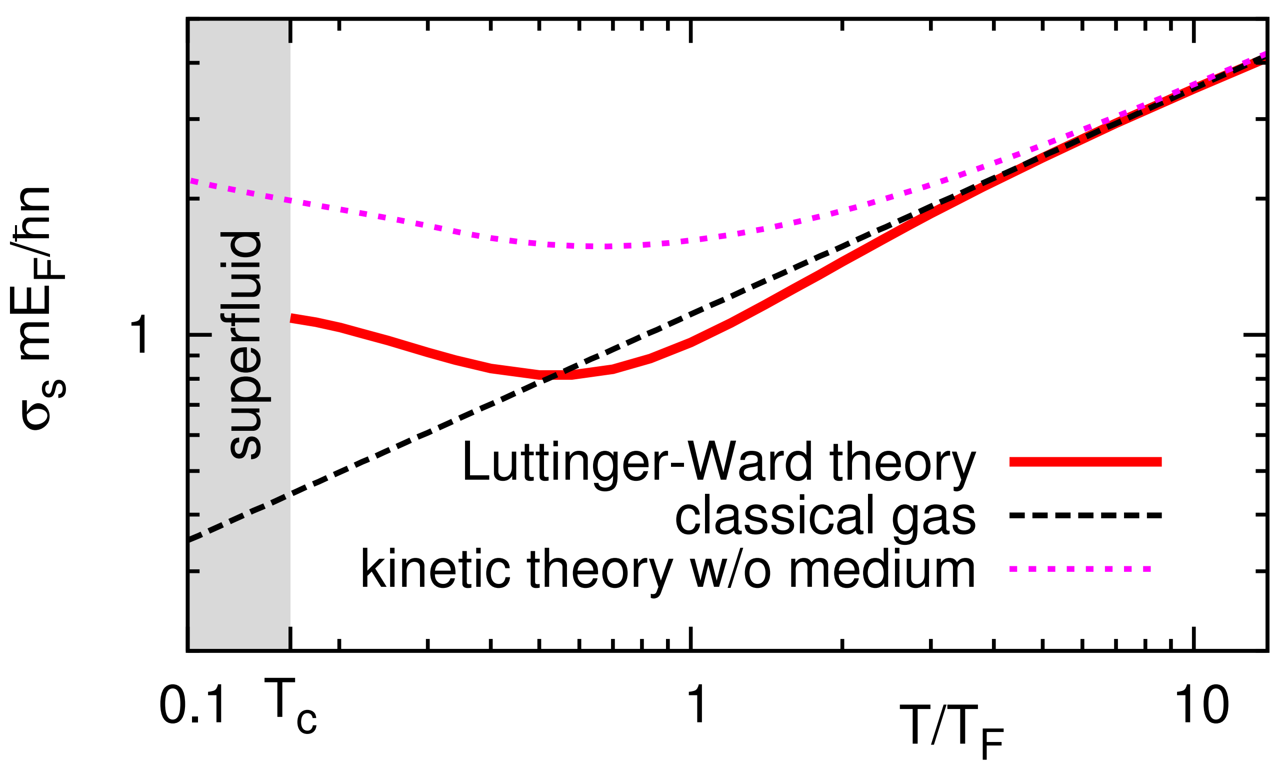
- scattering conserves total ↑+↓ momentum: mass current preserved
  but changes relative ↑-↓ momentum: spin current decays



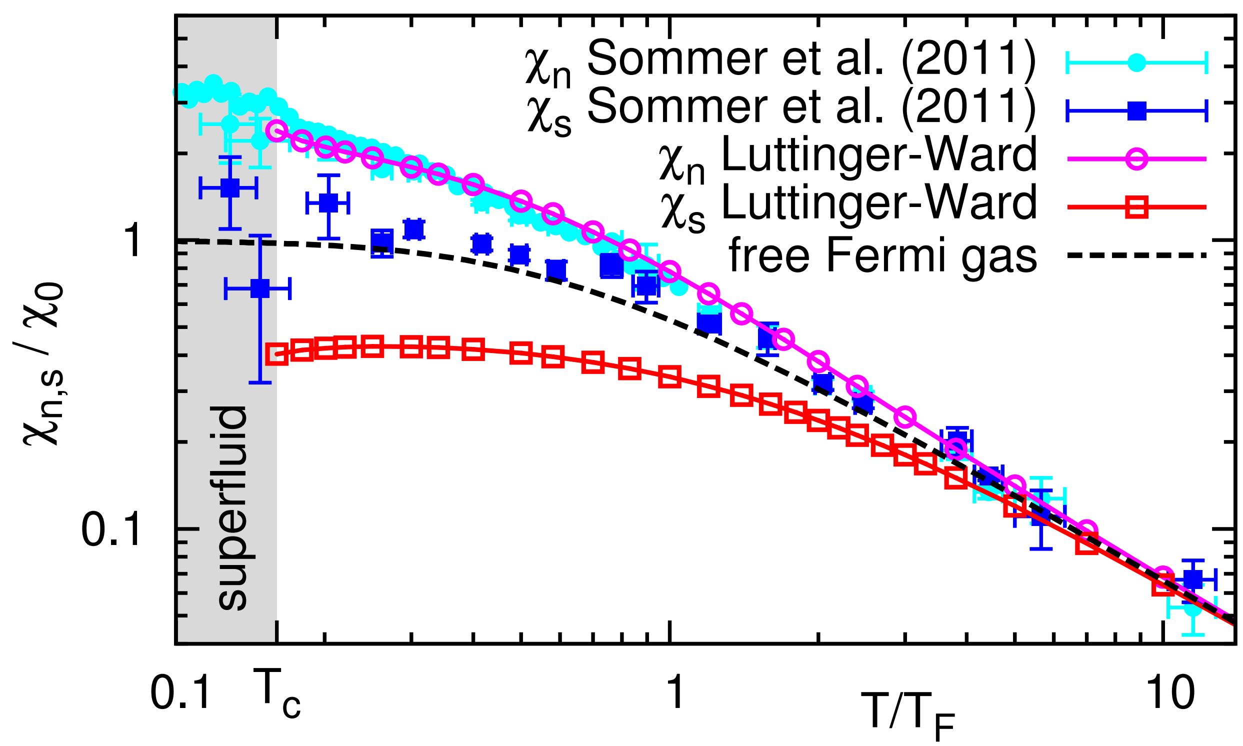
Spin conductivity and spin susceptibility

use Einstein relation $D_s = \frac{\sigma_s}{\chi_s}$

spin conductivity:



spin susceptibility:



Enss & Haussmann PRL 2012

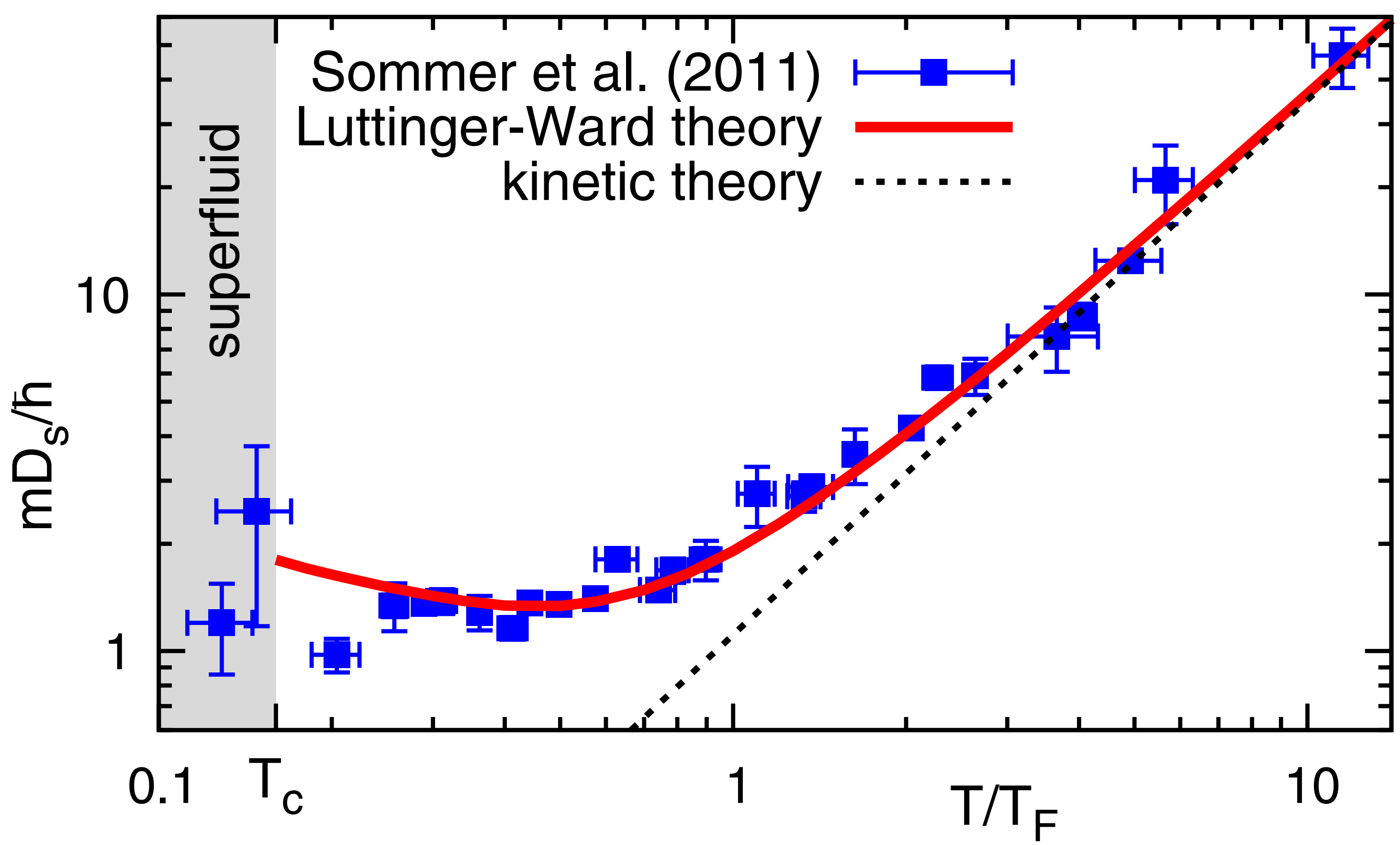
medium effects important:

- large-N transport calculation Enss PRA 2012
- in two dimensions Enss, Küppersbusch, Fritz PRA 2012

related work on spin transp.:
Bruun 2011; Duine et al. 2011;
Mink et al. 2012, 2013

Spin diffusivity

- obtain diffusivity from Einstein relation, $D_s = \frac{\sigma_s}{\chi_s}$



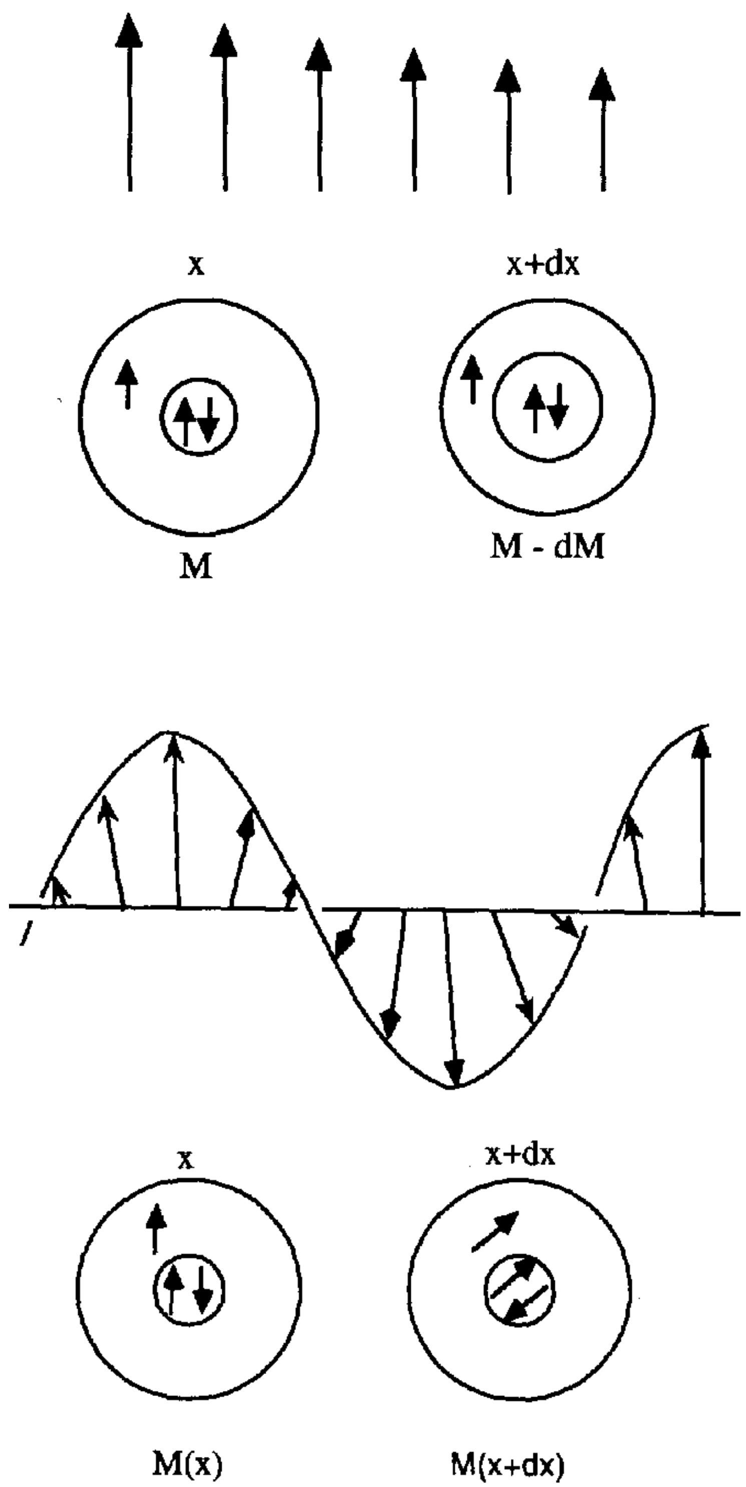
(experiment rescaled from trap to infinite homogeneous box)

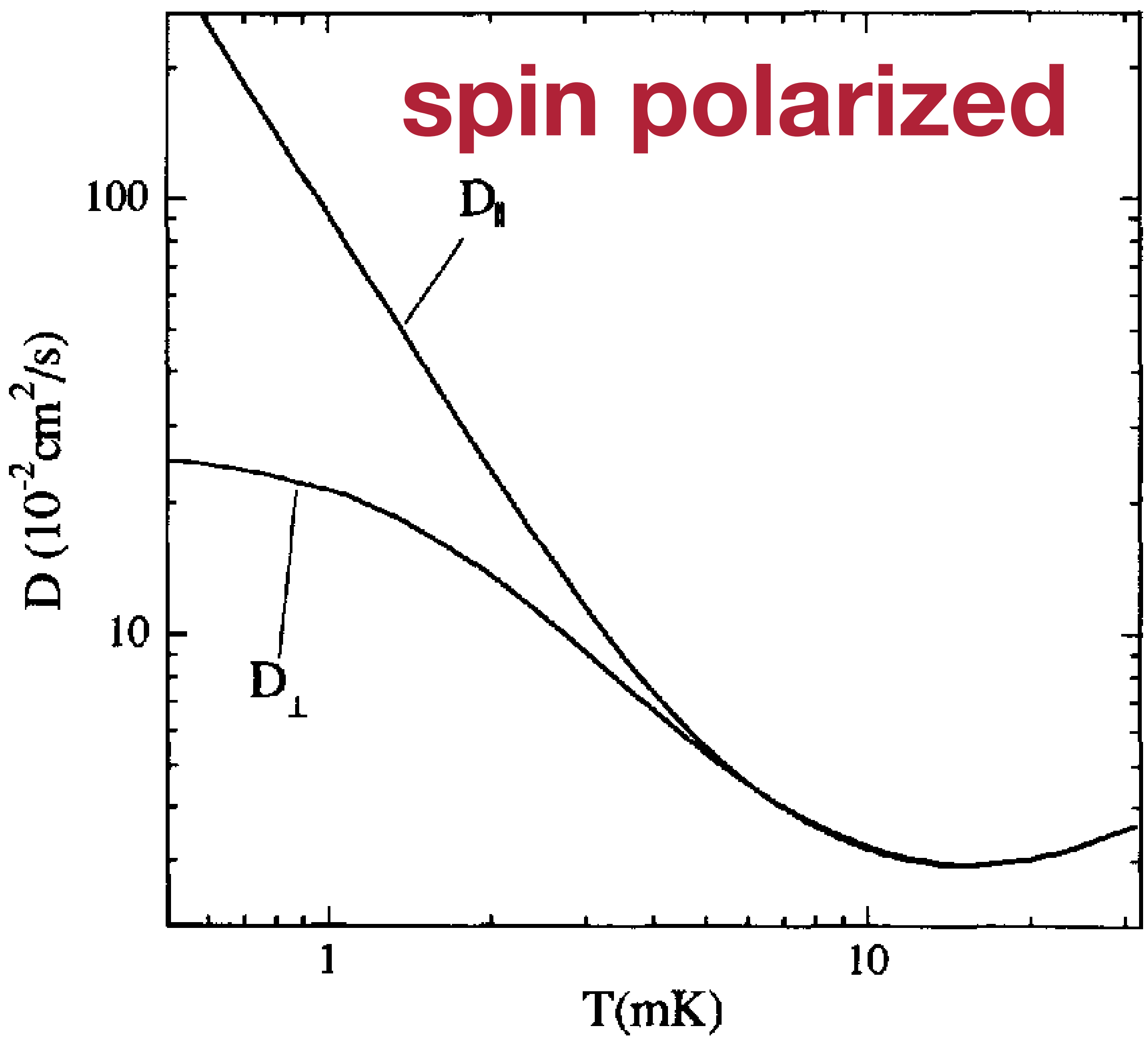
minimum $D_s \simeq 1.3 \frac{\hbar}{m}$

Enss & Haussmann PRL 2012

- Quantum Monte Carlo simulation for finite range interaction: $D_s \gtrsim 0.8 \frac{\hbar}{m}$
  Wlazlowski et al. PRL 2013

Longitudinal vs **transverse** spin diffusion

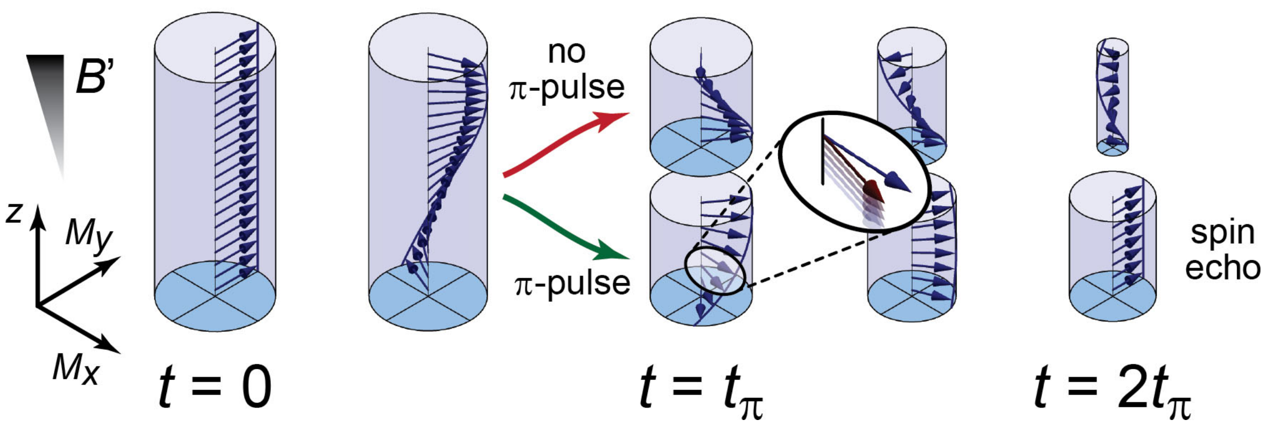


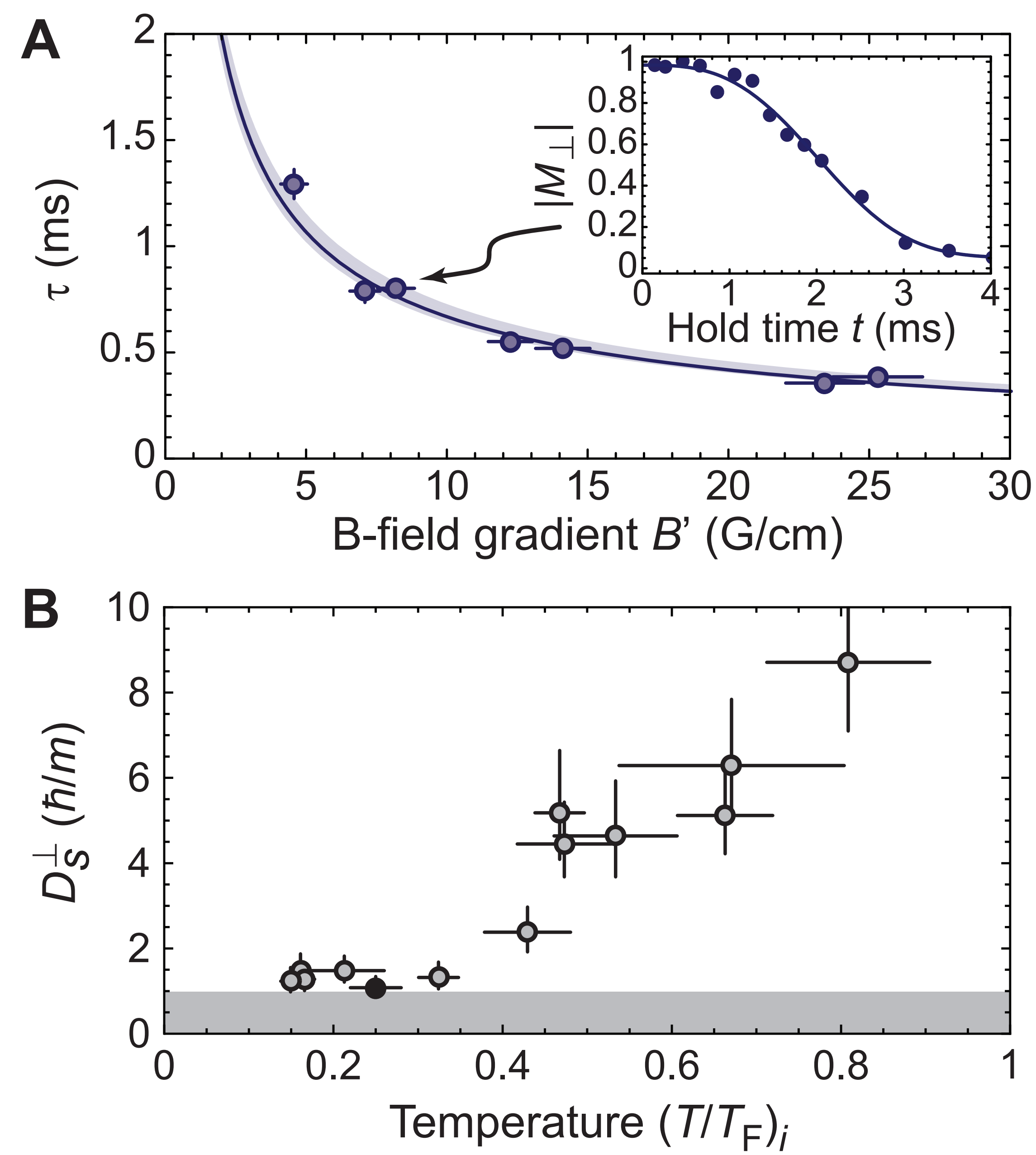


Mullin & Jeon 1992

Spin-echo experiment (Thywissen group, Toronto)

$$M_\perp(t) \;=\; -i\exp[-t^3/24\tau^3]$$





Bardon+ Science **334**, 722 (2014)

2D: Koschorreck et al. Nature Physics 2013

Spin diffusion in kinetic theory

- local magnetization vector and gradient

$$\boldsymbol{\mathcal{M}}(\boldsymbol{r},t) = \mathcal{M}(\boldsymbol{r},t)\,\hat{\boldsymbol{e}}(\boldsymbol{r},t) \qquad \frac{\partial \boldsymbol{\mathcal{M}}}{\partial r_i} = \frac{\partial \mathcal{M}}{\partial r_i}\hat{\boldsymbol{e}} + \mathcal{M}\frac{\partial \hat{\boldsymbol{e}}}{\partial r_i}$$

- Boltzmann equation for spin distribution function

longitudinal

$$\frac{D\boldsymbol{\sigma}_p}{Dt} \equiv \frac{\partial \boldsymbol{\sigma}_p}{\partial t} - \sum_i v_{pi}\frac{\partial \mathcal{M}}{\partial r_i}\hat{\boldsymbol{e}}\sum_\sigma t_\sigma \frac{\partial n_{p\sigma}}{\partial \varepsilon_p}$$

$$+ \sum_i v_{pi}\frac{\partial \hat{\boldsymbol{e}}}{\partial r_i}(n_{p+} - n_{p-}) + \boldsymbol{\Omega}\times\boldsymbol{\sigma}_p = \left(\frac{\partial \boldsymbol{\sigma}_p}{\partial t}\right)_{\text{coll}}$$

transverse **spin rotation**

Landau 1956, Silin 1957;
Leggett & Rice 1968-70;
Lhuillier & Laloë 1982;
Meyerovich 1985;
Jeon & Mullin 1988, 1992

- **many-body T-matrix** in collision integral and spin rotation Enss 2013
  derived as leading order in large-N expansion Enss 2012

Spin-rotation effect

- diffusion equation: Leggett 1970; Jeon & Mullin 1988; Enss 2013

$$M^+(\boldsymbol{r},t) = M_x + iM_y : \quad \frac{\partial M^+}{\partial t} \simeq -i\Omega_0(\boldsymbol{r})M^+ + D_\perp(1+i\mu M_z)\nabla^2 M^+$$

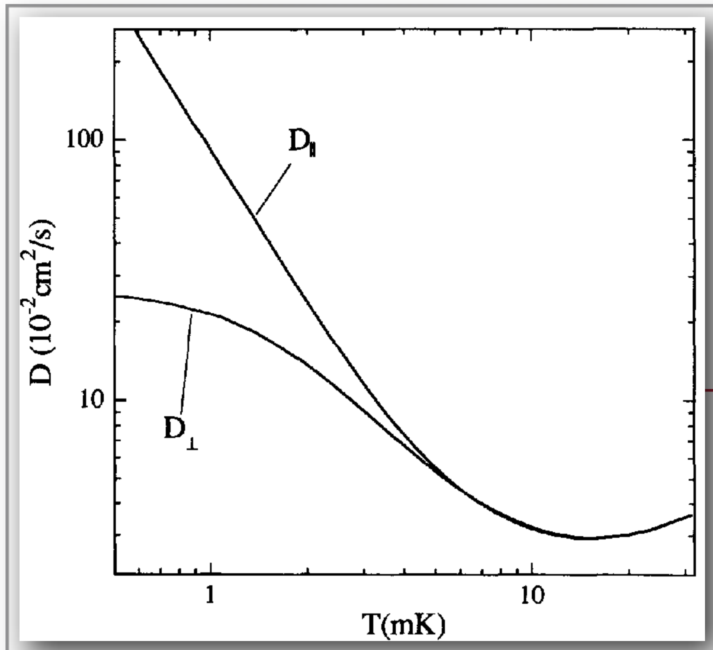
Leggett-Rice **spin-rotation effect:** complex diffusion constant, spin waves; spin current precesses around effective molecular field:

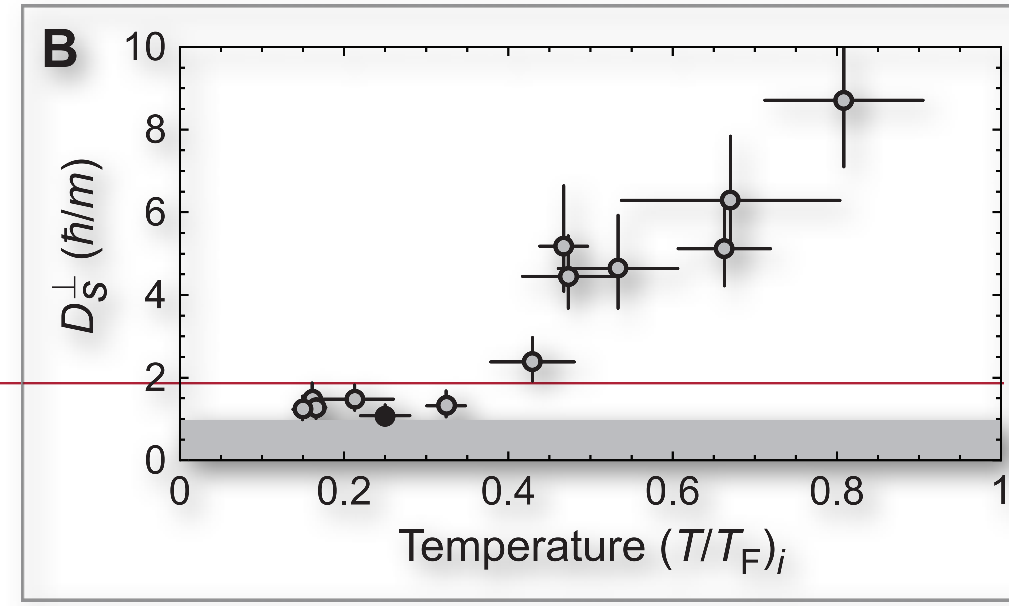
spin-rotation parameter

$$\mu = -\Omega_{\mathrm{mf}}\,\tau_\perp$$

Enss PRA 2013

n diffusivity (3D)





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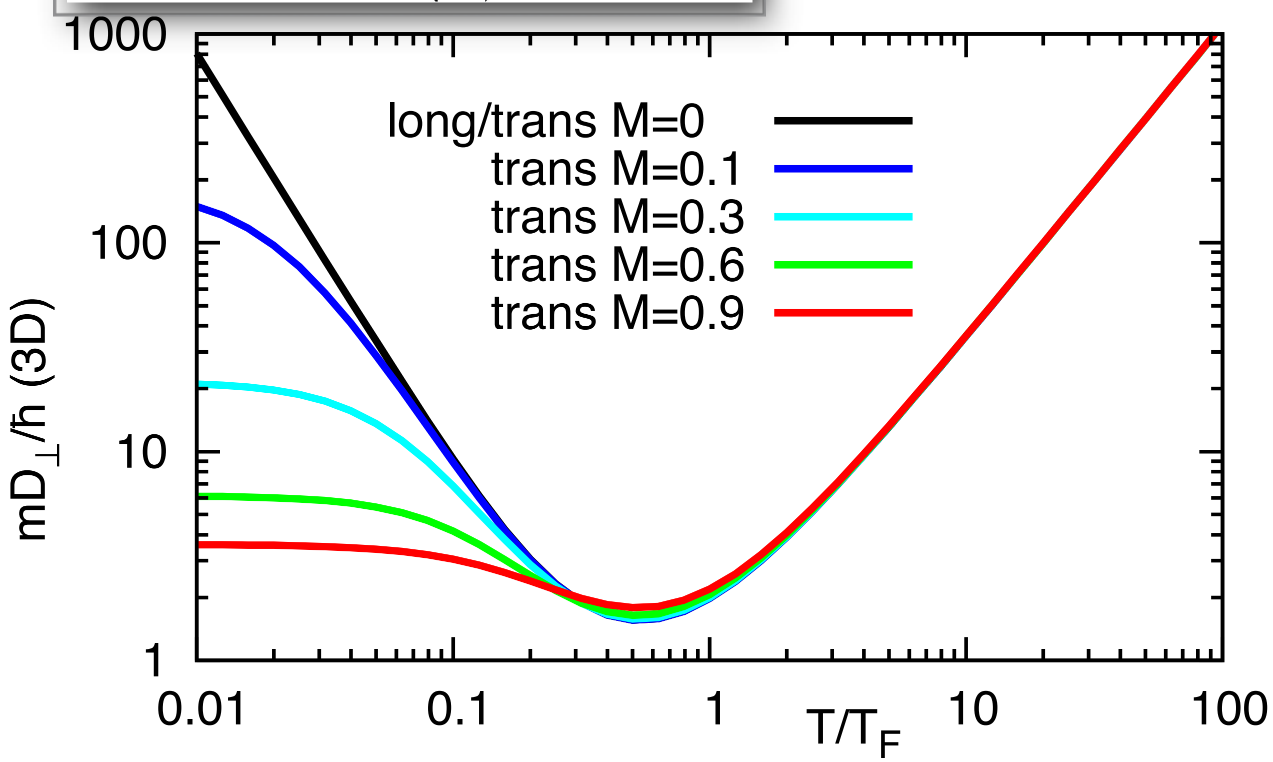
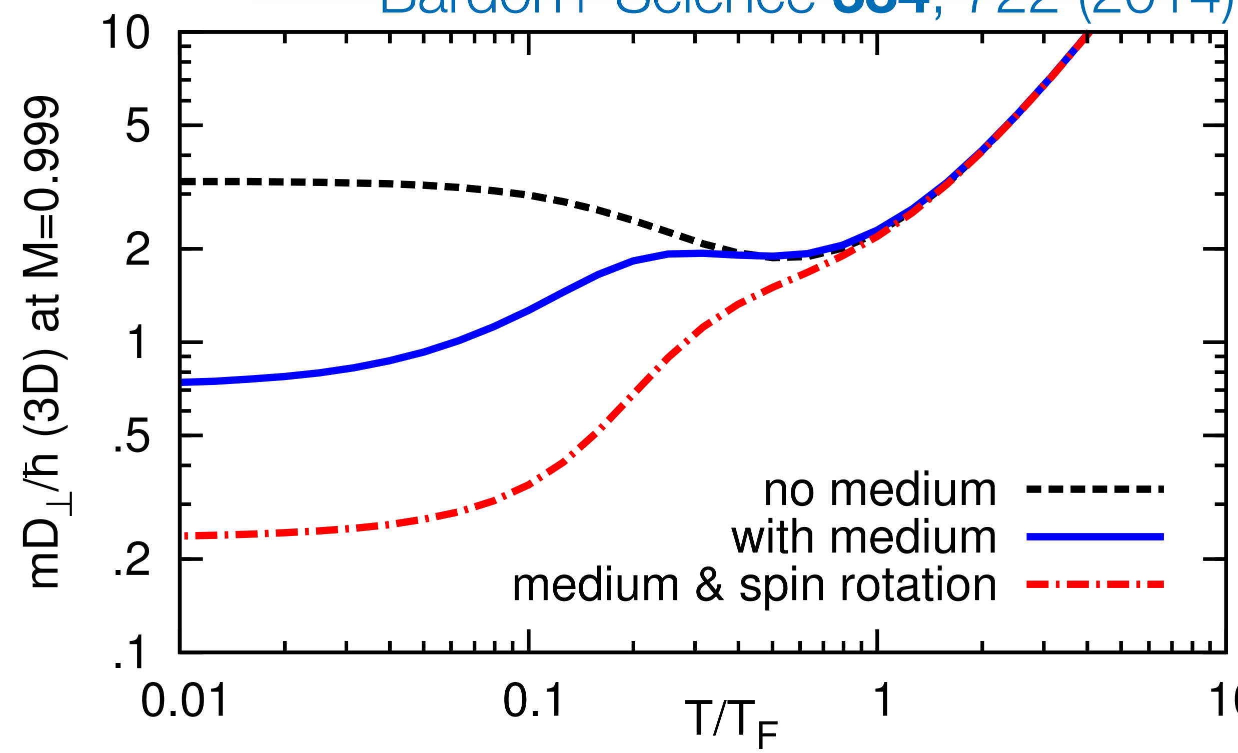


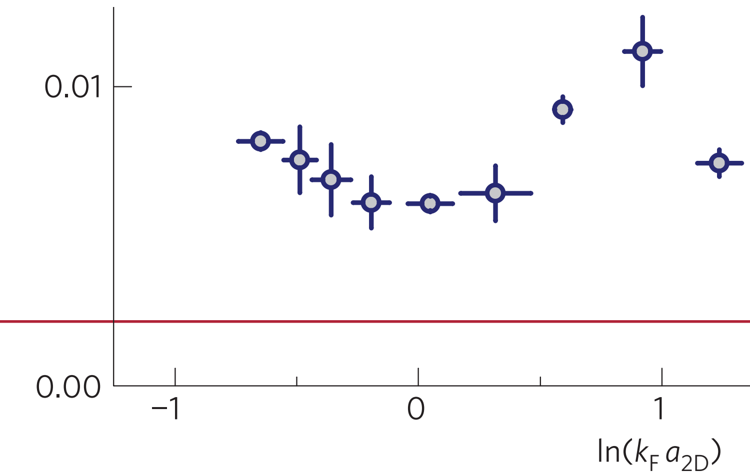


using vacuum (two-body) scattering cross section: similar to helium case

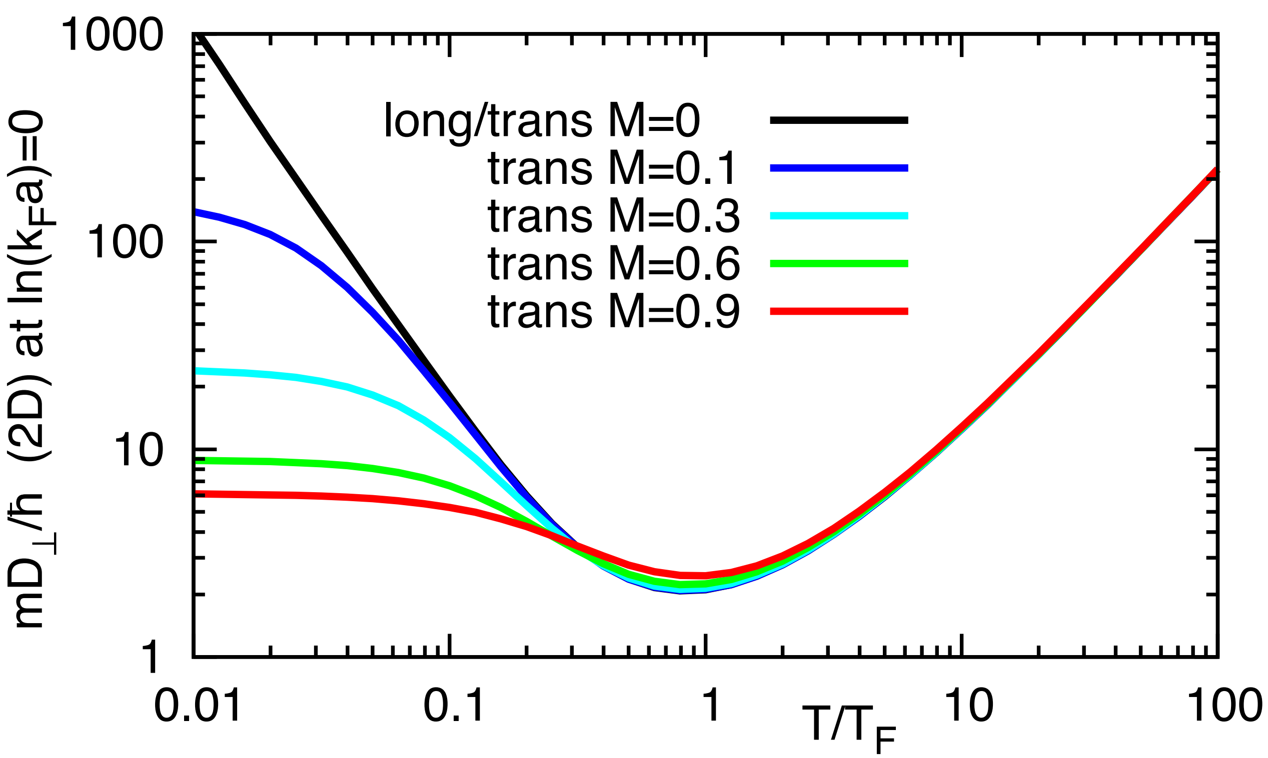
medium (finite density) scattering and spin-rotation effect: diffusivity much smaller!

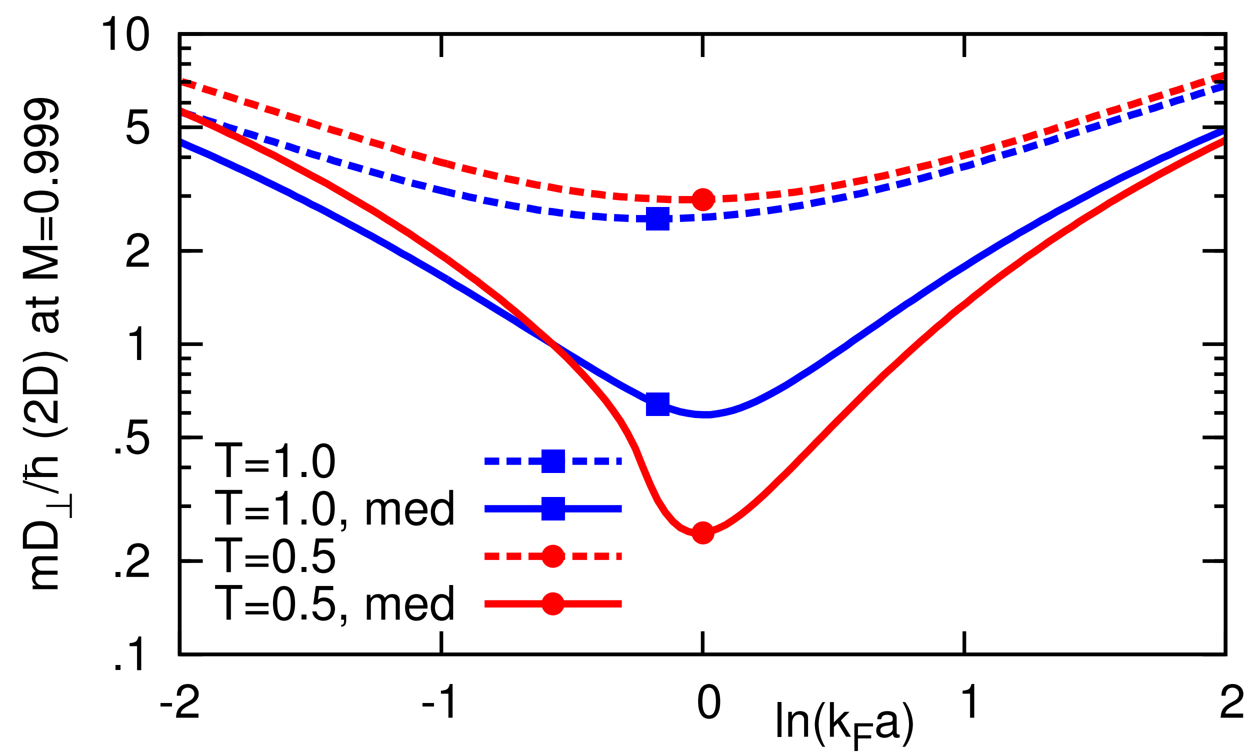
Enss PRA 2013

Transverse spin diffusivity (2D)



Koschorreck et al. 2013





vacuum scattering

Enss PRA 2013

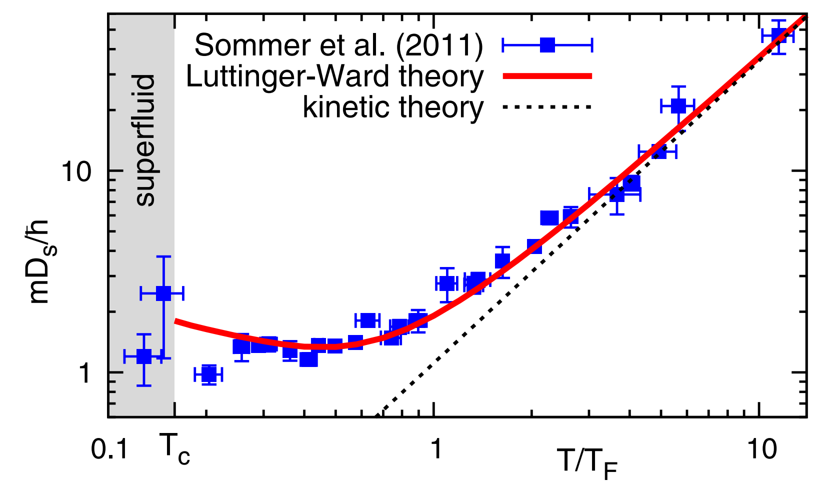
dependence on interaction and importance of medium effects

cf. η: Enss, Küppersbusch & Fritz PRA 2012

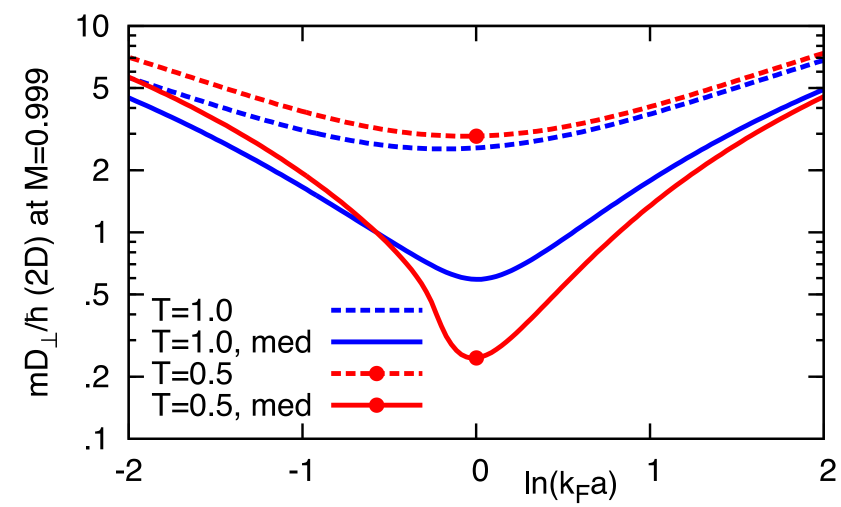
trap: Chiacchiera, Davesne, Enss & Urban, PRA 2013

Conclusion and outlook

- lowest friction at strong interaction: almost perfect fluidity near QCP
  Luttinger-Ward: Enss, Haussmann & Zwerger, Ann. Phys. **326**, 770 (2011)
  large-N: Enss, PRA **86**, 013616 (2012)



- quantitative understanding of spin diffusion:
  Luttinger-Ward transport calculation (tail, near Tc)
  slowest **longitudinal** spin diffusivity $D_s \gtrsim 1.3\,\hbar/m$
  Enss & Haussmann, PRL **109**, 195303 (2012)



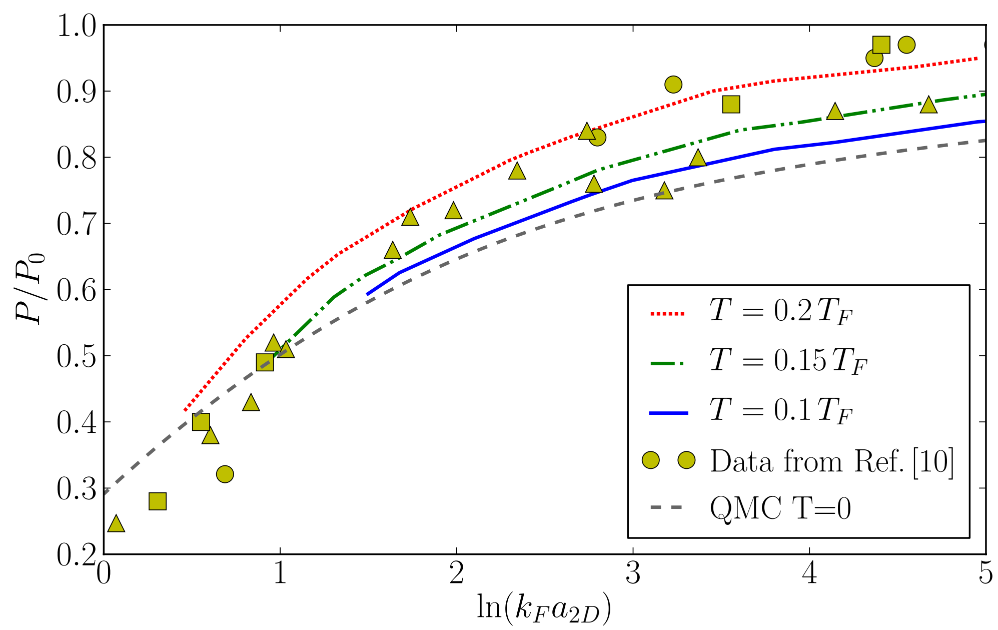
- transverse spin diffusion:
  $D_\perp$ can be much lower than $D_{||}$ in **degenerate, polarized** gas; Leggett-Rice **spin-rotation** effect
  Enss, PRA **88**, 033630 (2013)

- outlook: spin-rotation effect (Thywissen group)
  2D Fermi gas: EoS and pseudogap Bauer, Parish & Enss, PRL **112**, 135302 (2014)

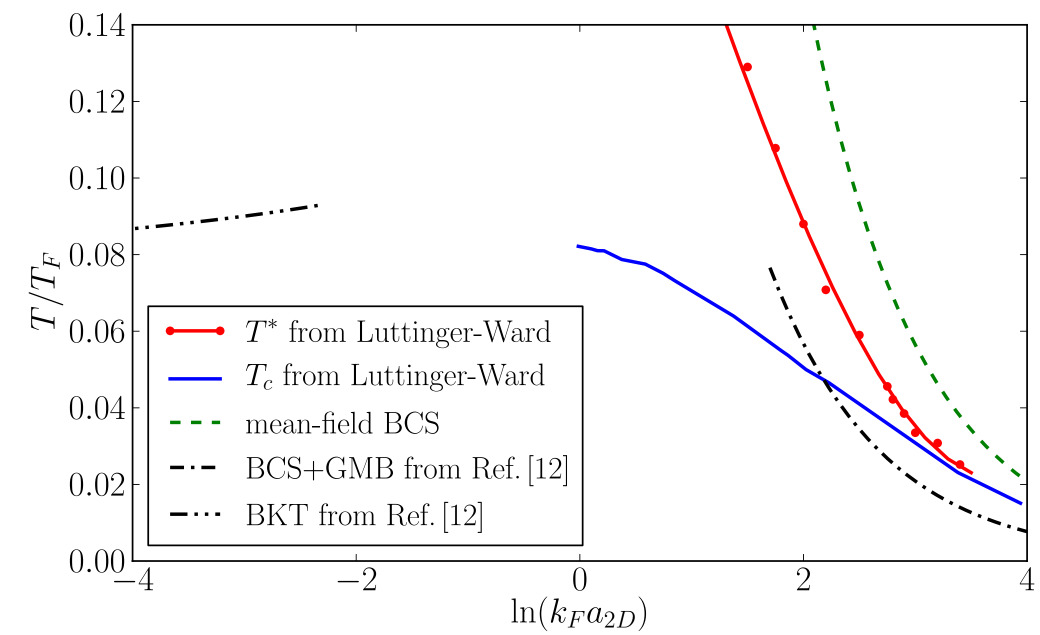
BKT-BCS crossover in 2D Fermi gas

Bauer, Parish & Enss,
PRL **112**, 135302 (2014)

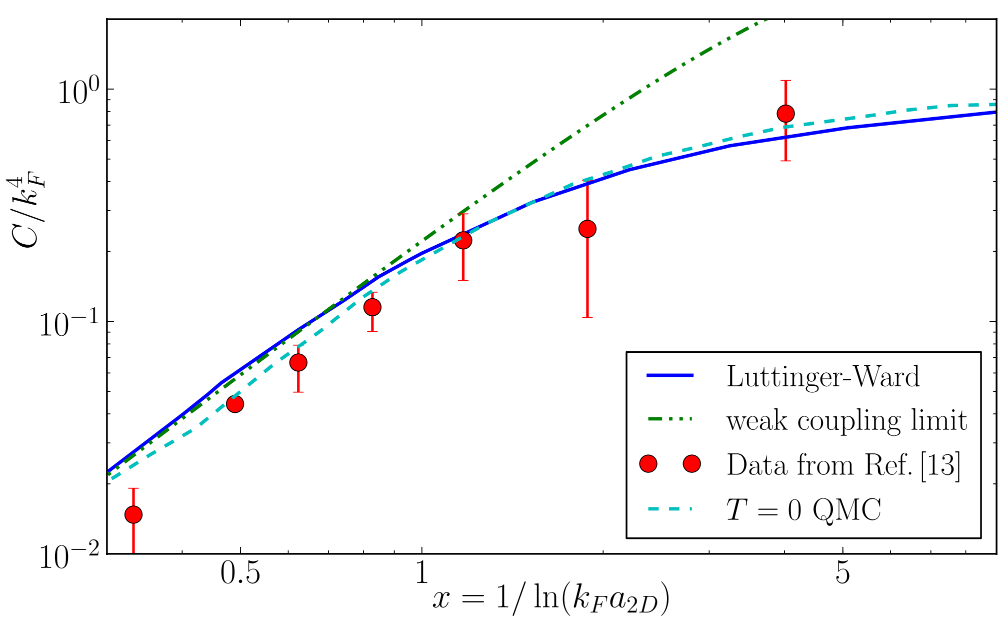
Pressure EoS (vs Turlapov data):



Phase diagram (Tc, T*):



Tan contact density (vs Köhl data):



Spectral function/pseudogap: